Recent status of leptohadron hypothesis

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Abstract

TGD predicts the existence of color octet excitations of ordinary leptons forming 'leptohadrons' as their color singlet bound states. There is some evidence on the existence of leptohadrons: the production of anomalous e^+e^- pairs in heavy ion collisions, Karmen anomaly, the anomalously high decay rate of ortopositronium (Op) and anomalous production of low energy e^+e^- pairs in hadronic collisions. PCAC makes it possible to predict the couplings of leptopion to leptons. The new contribution to Op decay rate is of correct order of magnitude and the anomaly allows to determine the precise value of the parameter f_{π_L}. Sigma model realization of PCAC makes it possible to construct a model for the production of leptohadrons in the electromagnetic fields of the colliding nuclei. π_L develops vacuum expectation value proportional to the 'instanton density' $E \cdot B$ and it is possible to relate leptohadron production rates to the Fourier transform of $E \cdot B$. Anomalous e^+e^- pairs must originate from $\sigma_L \pi_L$ pairs via $\sigma_L \rightarrow e^+e^-$ decay. The peculiar production characteristics of leptomesons are reproduced, the order of magnitude for the production cross section is correct and various decay rates are within experimental bounds. A resonance in photon photon scattering at cm energy equal to leptopion mass is predicted and leptobaryon pair production in heavy ion collisions is in principle possible. Leptopion contribution to the $\nu - e$ and $\bar{\nu} - e$ scattering should dominate at low energies.

1 Introduction

TGD suggest strongly ('predicts' is perhaps too strong expression) the existence of color excited leptons. The mass calculations based on p-adic thermodynamics and p-adic conformal invariance lead to a rather detailed picture about color excited leptons [Pitkänen$_c$].

a) Color excited leptons are color octets and leptohadrons are formed as their color singlet bound states.

b) The basic mass scale for leptohadron physics is completely fixed by the assumption that leptohadrons correspond to condensate level M_{127} (for the p-adic formulation of topological condensate concept see [Pitkänen$_b$]). Color excited leptons can have $k = 127, 113, 107, ...$ ($p \simeq 2^k$, k prime) condensation levels as primary condensation levels. The mass spectrum of leptohadrons is expected to have same general characteristics as hadronic mass spectrum and a satisfactory description should be based on string tension concept. The masses of ground state leptohadrons are calculable once primary condensation levels for colored leptons and the CKM matrix describing the mixing of color excited lepton families is known.

The strongest counter arguments against color excited leptons are the following ones:

a) The decay width of Z^0 boson allows only $N = 3$ light particles with neutrino quantum numbers. The introduction of new light elementary particles makes the decay width of Z^0 untolerably large. A purely TGD:eish solution of the problem was proposed on [Pitkänen$_c$] (5:th paper of series) and relied heavily on the relationship between p-adic and real probabilities.

b) The introduction of new colored states (also exotic quarks) spoils the asymptotic freedom of QCD. The proposed solution of problem was based on the idea that there is a different QCD associated with each Mersenne prime and these QCD:s do not communicate with each other. Also colored exotic bosons are predicted and these save the asymptotic freedom for each QCD.

One might stop the reading after these counterarguments unless there were definite experimental evidence supporting the leptohadron hypothesis.

a) The production of anomalous e^+e^- pairs in heavy ion collisions (energies just above the Coulomb barrier) suggests the existence of pseudoscalar particles decaying to e^+e^- pairs. In [Pitkänen and Mähonen] these states were

identified as leptopions that is bound states of color octet excitations of e^+ and e^-. The model for leptopion production was based on PCAC argument and led to an explanation for the peculiar production characteristics of leptopion. In [Pitkänen$_g$] the model was developed further by applying PCAC hypothesis. Unfortumately the calculations contained some (rather stupid) errors and led to too optimistic conclusions and partially erranous physical picture. For instance, the predicted production cross section was found to be of correct order of magnitude: unfortunately this was due to an error in numerical calculation.

b) The second puzzle, Karmen anomaly, is quite recent [KARMEN]. It has been found that in charge pion decay the distribution for the number of neutrinos accompanying muon in decay $\pi \rightarrow \mu + \nu_\mu$ as a function of time seems to have a small shoulder at $t_0 \sim ms$. A possible explanation is the decay of charged pion to muon plus some new weakly interacting particle with mass of order 30 MeV [Barger $et\ al$]: the production and decay of this particle would proceed via mixing with muon neutrino. TGD suggests the identification of this state as leptobaryon of type $L_B = f_{abc} L_8^a L_8^b \bar{L}_8^c$ having electroweak quantum numbers of neutrino. The mass of the exotic neutrino is indeed of correct order of magnitude (given by the muon mass scale).

c) The third puzzle is the anomalously high decay rate of ortopositronium. [Westbrook $et\ al$]. e^+e^- annihilation to virtual photon followed by the decay to real photon plus virtual leptopion followed by the decay of the virtual leptopion to real photon pair, $\pi_L \gamma \gamma$ coupling being determined by axial anomaly, provides a possible explanation of the puzzle.

d) There is also an anomalously large production of low energy e^+e^- pairs [Akesson $et\ al$, Barshay] in hadronic collisions, which might be basically due to the production of leptohadrons via the decay of virtual photons to colored leptons.

In this chapter a revised form of leptohadron hypothesis is described.

a) Sigma model realization of PCAC hypothesis allows to determine the decay widths of leptopion and leptosigma to e^+e^- pairs. Ortopositronium anomaly determines the value of f_{π_L} and therefore the value of leptopion-leptonucleon coupling and the decay rate of leptopion to two photons. Various decay widths are in accordance with experimental data and corrections to electroweak decay rates of neutron and muon are small. The resonances above 1.6 MeV are identified as string model satellite states of σ_L ('radial excita-

tions')

b) PCAC hypothesis and sigma model leads to a general model for lepto-hadron production in the electromagnetic fields of the colliding nuclei and production rates for leptopion and other leptohadrons are closely related to the Fourier transform of the instanton density $\bar{E} \cdot \bar{B}$ of the electromagnetic field created by nuclei. The most probable source of anomalous e^+e^- pairs is the production of $\sigma_L \pi_L$ pairs from vacuum followed by $\sigma_L \rightarrow e^+e^-$ decay. New effects are resonance in photon photon scattering at cm energy equal to leptopion mass and production of $e_{ex}\bar{e}_{ex}$ (e_{ex} is leptobaryon with quantum numbers of electron) and $e_{ex}\bar{e}$ pairs in heavy ion collisions.

c) Leptopion exchange gives dominating contribution to $\nu - e$ and $\bar{\nu} - e$ scattering at low energies and a new method of detecting solar neutrinos is proposed.

2 Leptohadron hypothesis

2.1 Anomalous e^+e^- pairs in heavy ion collisions

Heavy ion-collision experiments carried out at the Gesellschaft fur Schwerio-nenforschung in Darmstadt, West Germany [Schweppe *et al* , Clemente *et al* , Cowan *et al* , Tsertos *et al*] have yielded a rather puzzling set of results. The expectation was that in heavy ion collisions in which the combined charge of the two colliding ions exceeds 173, a composite nucleus with $Z > Z_{cr}$ would form and the probability for spontaneous positron emission would become appreciable.

Indeed, narrow peaks of widths of rougly 50-70 keV and energies about 350 ± 50 keV were observed in the positron spectra but it turned out that the position of peaks seems to be a constant function of Z rather that vary as Z^{20} as expected and that peaks are generated also for Z smaller than the critical Z. The collision energies at which peaks occur lie in the neighbourhodd of 5.7-6 MeV/nucleon. Also it was found that positrons are accompanied by e^-- emission. Data are consistent with the assumption that some structure at rest in cm is formed and decays subsequently to e^+e^- pair.

Various theoretical explanations for these peaks have been suggested [Chodos, Kraus and Zeller]. For example, lines might be created by pair

conversion in the presence of heavy nuclei. In nuclear physics explanations the lines are due to some nuclear transition that occurs in the compound nucleus formed in the collision or in the fragmets formed. The Z-independence of the peaks seems however to exclude both atomic and nuclear physics explanations [Chodos]. Elementary particle physics explanations [Chodos, Kraus and Zeller] seem to be excluded already by the fact that several peaks have been observed in the range $1.6 - 1.8\ MeV$ with widths of order $10 - 10^2\ keV$. These states decay to e^+e^- pairs. There is evidence for one narrow peak with width of order one keV at 1.062 Mev [Chodos]: this state decays to photon-photon pairs.

Thus it seems that the structures produced might be composite, perhaps resonances in e^+e^- system. The difficulty of this explanation is that conventional QED seems to offer no natural explanation for the strong force needed to explain the energy scale of the states. One idea is that the strong electromagnetic fields create a new phase of QED [Chodos] and that the resonances are analogous to pseudoscalar mesons appearing as resonances in strongly interacting systems.

TGD explanation proposed is based on the following hypothesis motivated by Topological Geometrodynamics [Pitkänen$_a$, Pitkänen$_b$].
a) Ordinary leptons are not point like particles and can have colored excitations, which form color singlet bound states. A natural identification for the primary condensate level is M_{127} so that the mass scale is of order one MeV for states containing lowest generation colored leptons.
b) The states in question are leptohadrons that is color confined states formed from the colored excitations of e^+ and e^-. $m = 1.062\ MeV$ state is identified as leptopion π_L and $m = 1.8\ MeV$ state turns out to be identifiable as scalar particle σ_L predicted by sigma model providing a realization of PCAC hypothesis. The remaining resonances can be identified in string model picture as $J = 0$ satellites of $\sigma_L(1.8\ MeV)$.

The program of the section is following:
a) PCAC hypothesis successfull in low energy pion physics is generalized to the case of leptopion. Hypothesis allows to deduce the coupling of leptopion to leptons and leptobaryons in terms of leptobaryon-lepton mixing angles. Ortopositronium anomaly allows to deduce precise value of f_{π_L} so that the crucial parameters of the model are completely fixed. The decay rates of

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leptopion to photon pair and of leptosigma to ordinary e^+e^- pairs are within experimental bounds and corrections to muon and beta decay rates are small. A new calculable resonance contribution to photon-photon scattering at cm energy equal to leptopion mass is predicted.

b) A model for for the leptohadron and leptopion production is constructed. The starting point is sigma model providing a realization of PCAC hypothesis. In an external electromagnetic field leptopion develops a vacuum expectation value proportional to electromagnetic anomaly term [Iztykson and Zuber] so that production amplitude for leptopion is essentially the Fourier transform of the scalar product of the electric field of the stationary target nucleus with the magnetic field of the colliding nucleus. Sigma model makes it possible to relate the production amplitude for $\sigma_L \pi_L$ pairs to the leptopion production amplitude: the key element of the model is the large value of the $\sigma \pi_L \pi_L$ coupling constant. The decays of the scalar particle σ_L (1.8 MeV state) and its radial excitations directly to e^+e^- states is the simplest explanation for the production of e^+e^- pairs. The fact that two-particle states are produced could perhaps explain the observed deviations from the simple resonance decay picture. Model predicts also a direct production of leptobaryon (e_{ex}) pairs in heavy ion collisions.

c) Leptohadron production amplitudes are proportional to leptopion production amplitude and this motivates a detailed study of leptopion production. Two models for leptopion production are developed: in classical model colliding nucleus is treated classically whereas in quantum model the colliding nucleus is described quantum mechanically. It turns out that classical model explains the peculiar production characteristics of leptopion but that production cross section is too small by several orders of magnitude. Quantum mechanical model predicts also diffractive effects: production cross section varies rapidly as a function of the scattering angle and for a fixed value of scattering angle there is a rapid variation with the collision velocity. The estimate for the total e^+e^- production cross section is of correct order of magnitude due to the coherent summation of the contributions to the amplitude from different values of the impact parameter at the peak.

c) The problem of understanding the effective experimental absence of leptohadronic color interactions in TGD picture is discussed.

2.2 Leptopions and generalized PCAC hypothesis

One can say that the PCAC hypothesis predicts the existence of pions and a connection between the pion nucleon coupling strength and the pion decay rate to leptons. In the following we give the PCAC argument and its generalization and consider various consequences.

2.2.1 PCAC for ordinary pions

The PCAC argument for ordinary pions goes as follows [Okun]:
a) Consider the contribution of the hadronic axial current to the matrix element describing lepton nucleon scattering (say $N + \nu \rightarrow P + e^-$) by weak interactions. The contribution in question reduces to the well-known current-current form

$$
\begin{aligned}
M &= \frac{G_F}{\sqrt{2}} g_A L_\alpha \langle P|A^\alpha|P \rangle \\
L_\alpha &= \bar{e}\gamma_\alpha(1 + \gamma_5)\nu \\
\langle P|A^\alpha|P \rangle &= \bar{P}\gamma^\alpha N
\end{aligned}
\tag{1}
$$

where $G_F = \frac{\pi\alpha}{2m_W^2 \sin^2(\theta_W)} \simeq 10^{-5}/m_p^2$ denotes the dimensional weak interaction coupling strength and g_A is the nucleon axial form factor:$g_A \simeq 1.253$.
b) The matrix element of the hadronic axial current is not divergenceless, due to the nonvanishing nucleon mass,

$$
a_\alpha \langle P|A^\alpha|P \rangle \simeq 2m_p \bar{P}\gamma_5 N
\tag{2}
$$

Here q^α denotes the momentum transfer vector. In order to obtain divergenceless current, one can modify the expression for the matrix element of the axial current

$$
\langle P|A^\alpha|N \rangle \rightarrow \langle P|A^\alpha|N \rangle - q^\alpha 2m_p \bar{P}\gamma_5 N \frac{1}{q^2}
\tag{3}
$$

c) The modification introduces a new term to the lepton-hadron scattering amplitude identifiable as an exchange of a massless pseudoscalar particle

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$$\delta T \;=\; \frac{G_F g_A}{\sqrt{2}} L_\alpha \frac{2m_p q^\alpha}{q^2} \bar{P}\gamma_5 N \qquad (4)$$

The amplitude is identifiable as the amplitude describing the exchange of the pion, which gets its mass via the breaking of chiral invariance and one obtains by the straightfowread replacement $q^2 \to q^2 - m_\pi^2$ the correct form of the amplitude.

d) The nontrivial point is that the interpretations as pion exhange is indeed possible since the amplitude obtained is to a good approximation identical to that obtained from the Feynman diagram describing pion exchange, where the pion nucleon coupling constant and pion decay amplitude appear

$$T_2 \;=\; \frac{G}{\sqrt{2}} f_\pi q^\alpha L_\alpha \frac{1}{q^2 - m_\pi^2} g\sqrt{2}\bar{P}\gamma_5 N \qquad (5)$$

The condition $\delta T \sim T_2$ gives from Goldberger-Treiman [Okun]

$$g_A(\simeq 1.25) \;=\; \sqrt{2}\frac{f_\pi g}{2m_p}(\simeq 1.3) \qquad (6)$$

satisfied in a good accuracy experimentally.

2.2.2 PCAC in leptonic sector

A natural question is why not generalize the previous argument to the leptonic sector and look at what one obtains. The generalization is based on following general picture.

a) There are two levels to be considered: the level of ordinary leptons and the level of leptobaryons of type $f_{ABC}L_8^A L_8^B \bar{L}_8^C$ possessing same quantum numbers as leptons. The interaction transforming these states to each other causes in mass eigenstates mixing of leptobaryons with ordinary leptons described by mixing angles. The masses of lepton and corresponding leptobaryon could be quite near to each other and in case of electron should be the case as it turns out.

b) A counterargument against the applications of PCAC hypothesis at level of ordinary leptons is that baryons and mesons are both bound states of

quarks whereas ordinary leptons are not bound states of color octet leptons. The divergence of the axial current is however completely independent of the possible internal structure of leptons and microscopic emission mechanism. Ordinary lepton cannot emit leptopion directly but must first transform to leptobaryon with same quantum numbers: phenomenologically this process can be described using mixing angle $sin(\theta_B)$. The emission of leptopion proceeds as $L \to B_L$: $B_L \to B_L + \pi_L$: $B_L \to L$, where B_L denotes leptobaryon of type structure $f_{ABC} L_8^A L_8^B \bar{L}_8^C$. The transformation amplitude $L \to B_L$ is proportional to the mixing angle $sin(\theta_L)$.

Three different PCAC type identities are assumed to hold true:

PCAC1) The vertex for the emission of leptopion by ordinary lepton is equivalent with the graph in which lepton L transforms to leptobaryon L^{ex} with same quantum numbers, emits leptopion and transforms back to ordinary lepton. The assumption relates the couplings $g(L_1, L_2)$ and $g(L_1^{ex}, L_2^{ex})$ (analogous to strong coupling) and mixing angles to each other

$$g(L_1, L_2) \;=\; g(L_1^{ex}, L_2^{ex}) sin(\theta_1) sin(\theta_2) \tag{7}$$

The condition implies that in electroweak interactions ordinary leptons do not transform to their exotic counterparts.

PCAC2) The generalization of the ordinary Goldberger-Treiman argument holds true, when ordinary baryons are replaced with leptobaryons. This gives the condition expressing the coupling $f(\pi_L)$ of the leptopion state to axial current defined as

$$\langle vac | A_\alpha | \pi_L \rangle = i p_\alpha f_{\pi_L} \tag{8}$$

in terms of the masses of leptobaryons and strong coupling g.

$$f_{\pi_L} \;=\; \sqrt{2} g_A \frac{(m_{ex}(1) + m_{ex}(2)) sin(\theta_1) sin(\theta_2)}{g(L_1, L_2)} \tag{9}$$

where g_A is parameter characterizing the deviation of weak coupling strength associated with leptobaryon from ideal value: $g_A \sim 1$ holds true in good approximation.

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PCAC3) The elimination of leptonic axial anomaly from leptonic current fixes the values of $g(L_i, L_j)$.

i) The standard contribution to the scattering of leptons by weak interactions.sis given by the expression

$$T = \frac{G_F}{\sqrt{2}}\langle L_1|A^\alpha|L_2\rangle\langle L_3|A_\alpha|L_4\rangle$$
$$\langle L_i|A^\alpha|L_j\rangle = \bar{L}_i\gamma^\alpha\gamma_5 L_j \tag{10}$$

ii) The elimination of the leptonic axial anomaly

$$q_\alpha\langle L_i|A^\alpha|L_j\rangle = (m(L_i) + m(L_j))\bar{L}_i\gamma_5 L_j \tag{11}$$

by modifying the axial current by the anomaly term

$$\langle L_i|A^\alpha|L_j\rangle \rightarrow \langle L_i|A^\alpha|L_j\rangle - (m(L_i) + m(L_j))\frac{q^\alpha}{q^2}\bar{L}_i\gamma_5 L_j \tag{12}$$

induces a new interaction term in the scattering of ordinary leptons.

iii) It is assumed that this term is equivalent with the exchange of leptopion. This fixes the value of the coupling constant $g(L_1, L_2)$ to

$$g(L_1, L_2) = 2^{1/4}\sqrt{G_F}(m(L_1) + m(L_2))\xi$$
$$\xi(charged) = 1$$
$$\xi(neutral) = cos(\theta_W) \tag{13}$$

Here the coefficient ξ is related to different values of masses for gauge bosons W and Z appearing in charged and neutral current interactions. An important factor 2 comes from the modification of the axial current in both matrix elements of the axial current.

Leptopion exchange interaction couples right and left handed leptons to each other and its strength is of the same order of magnitude as the strength of the ordinary weak interaction at energies not considerably large than the mass of the leptopion. At high energies this interaction is negligible and

the existence of the leptopion predicts no corrections to the parameters of the standard model since these are determined from weak interactions at much higher energies. If leptopion mass is sufficiently small (as found, $m(\pi_L) < 2m_e$ is allowed by the experimental data), the interaction mediated by leptopion exchange can become quite strong due to the presence of the leptopion progator. The value of the lepton leptopion coupling is $g(e, e) \equiv g \sim 5.6 \cdot 10^{-6}$. It is perhaps worth noticing that the value of the coupling constant is of the same order as lepton-Higgs coupling constant and also proportional to the mass of the lepton. This is accordance with the idea that the components of the Higgs boson correspond to the divergences of various vector currents [Pitkänen$_a$]. What is important that the value of the coupling is completely independent of the details of leptopion emission.

PCAC identities fix the values of coupling constants apart from the values of mixing angles. If one assumes that the strong interaction mediated by leptopions is really strong and the coupling strength $g(L_{ex}, L_{ex})$ is of same order of magnitude as the ordinary pion nucleon coupling strength $g(\pi NN) \simeq 13.5$ one obtains an estimate for the value of the mixing angle $sin(\theta_e)$ $sin(\theta_e) \sim \frac{g(\pi NN)}{g(L,L)} \sim 2.4 \cdot 10^{-6}$. This implies the order of magnitude $10^{-11} m_W \sim eV$ for f_{π_L}. The experimental bound for $\pi_L \to \gamma\gamma$ decay width implies that $sin(\theta_e)$ must be at least by a factor $10^{5/4}$ larger. Ortopositronium decay rate anomaly $\Delta\Gamma/\Gamma \sim 10^{-3}$ and the assumption $m_{ex} \geq 1.3~MeV$ (so that $e_{ex}\bar{e}$ decay is not possible) gives the upper bound $sin(\theta_e) \leq x \cdot 10^{-4}$, where the value of $x \sim 1$ depends on the number of leptopion type states and on the precise value of Op anomaly. Leptohadronic strong coupling satisfies $g \leq g_{max} \sim 1$.

2.3   Leptopion decays and PCAC hypothesis

The PCAC argument makes it possible to predict the leptopion coupling and decay rates of leptopion to various channels. Actually the orders of magnitude for the decay rates of leptosigma and other leptomesons can be deduced also. The comparison with the experimental data is made difficult by the uncertainty of the identifications. The lightest candidate has mass $1.062~MeV$ and decay width of order 1 keV [Chodos]: only photon photon decay has been observed for this state. The next leptomeson candidates are in the mass range $1.6 - 1.8~MeV$. Perhaps the best status is possessed by 'Darmstadtium' with mass $1.8~MeV$. For these states decays final states

identified as e^+e^- pairs dominate: these states probably correspond to leptosigma and its string model satellites ('radial excitations'). Hadron physics experience suggests that the decay widths of the leptohadrons (leptopion forming a possible exception) should be about 1-10 per cent of particle mass as in hadron physics. The upper bounds for the widths are indeed in the range $50 - 70\ keV$ [Chodos].

a) As in the case of the ordinary pion, anomaly considerations give the following approximate expression for the decay rate of the leptopion to two-photon final states [Iztykson and Zuber])

$$\Gamma(\pi_L \to \gamma\gamma) \quad = \quad \frac{\alpha^2 m^3(\pi_L)}{64 f_{\pi_L}^2 \pi^3} = \left(\frac{m(\pi_L)}{m(\pi)}\right)^3 \frac{f_\pi^2}{f_{\pi_L}^2} \Gamma(\pi \to \gamma\gamma) \qquad (14)$$

where the decay rate for the ordinary pion is given by $\Gamma(\pi \to \gamma\gamma) \simeq 1.5 \cdot 10^{-16}\ sec$. For $m(\pi_L) = 1.062\ MeV$ and $f_{\pi_L} = 7.9\ keV$ implied by the ortopositronium decay rate anomaly $\Delta\Gamma/\Gamma = 10^{-3}$ one has $\Gamma(\gamma\gamma) = .52\ keV$, which is consistent with the experimental estimate of order $1\ keV$ [Chodos]. Actually several leptopion states (string model satellites) could exist in one-one correspondence with σ scalars (3 at least). Since all 3 leptopion states contribute to Op decay rate, the actual value of f_{π_L} assumed to scale as $m(\pi_L)$, is actually larger in this case: it turns out that f_{π_L} for the lightest leptopion increases to $f_{\pi_L} = 11\ keV$ and gives $\Gamma(\gamma\gamma) \simeq .27\ keV$. The increase of the ortopositronium anomaly by a factor of, say 4, implies corresponding decrease in $f_{\pi_L}^2$. The value of $f_{\pi_L}^2$ is also sensitive to the precise value of the mass of the lightest leptopion. The production cross section for anomalous e^+e^- pairs turns out to be proportional to $f_{\pi_L}^{-4}$ and is very sensitive to the exact value of f_{π_L}: $f_{\pi_L} \sim 1\ keV$ is favoured and corresponds to Op anomaly $\Delta\Gamma/\Gamma \sim 4 \cdot 10^{-3}$ in 3-leptopion case.

b) The value of the leptopion-lepton coupling can be used to predict the decay rate of leptopion to leptons. One obtains for the decay rate $\pi_L^0 \to e^+e^-$ the estimate

$$\Gamma(\pi_L \to e^+e^-) \quad = \quad 4\frac{g(e,e)^2\pi}{2(2\pi)^2}(1 - 4x^2)m(\pi_L)$$

$$= \quad 16Gm_e^2 cos^2(\theta_W)\frac{\sqrt{2}}{4\pi}(1 - 4x^2)m(\pi_L)$$

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$$x = \frac{m_e}{m(\pi_L)} \tag{15}$$

for the decay rate of the leptopion: for leptopion mass $m(\pi_L) \simeq 1.062\ MeV$ one obtains for the decay rate the estimate $\Gamma \sim 1/(1.3 \cdot 10^{-8}\ sec)$: the low decay rate is partly due to the phase space suppresion and implies that e^+e^- decay products cannot be observed in the measurement volume. The low decay rate is in accordance with the identification of the leptopion as the $m = 1.062\ MeV$ leptopion candidate. In sigma model leptopion and leptosigma have identical lifetimes and for leptosigma mass of order $1.8\ MeV$ one obtains $\Gamma(\sigma_L \to e^+e^-) \simeq 1/(8.2 \cdot 10^{-10}\ sec)$: the prediction is larger than the lower limit $\sim 1/(10^{-9}\ sec)$ for the decay rate implied by the requirement that σ_L decays inside the measurement volume. The estimates of the lifetime obtained from heavy ion collisions [Koenig $et\ al$] give the estimate $\tau \geq 10^{-10}\ sec$. The large value of the lifetime is in accordance with the limits for the lifetime obtained from Babbha scattering [Judge $et\ al$], which indicate that the lifetime must be longer than 10^{-12} sec.

For leptomeson candidates with mass above $1.6\ MeV$ no experimental evidence for other decay modes than $X \to e^+e^-$ has been found and the empirical upper limit for $\gamma\gamma/e^+e^-$ branching ratio [Dantzman $et\ al$] is $\Gamma(\gamma\gamma)/\Gamma(e^+e^-) \leq 10^{-3}$. If the identification of the decay products as e^+e^- pairs (rather than $e_{ex}\bar{e}_{ex}$ pairs of leptonucleons!) is correct then the only possible conclusion is that these states cannot correspond to leptopion since leptopion should decay dominantly into photon photon pairs.

c) The expression for the decay rate $\pi_L \to e + \bar{\nu}_e$ reads as

$$\begin{aligned}
\Gamma(\pi_L^- \to e\nu_e) &= 8Gm_e^2 \frac{(1-x^2)^2}{2(1+x^2)} \frac{\sqrt{2}}{(2\pi)^5} m(\pi_L) \\
&= \frac{4}{cos^2(\theta_W)} \frac{(1-x^2)}{(1+x^2)(1-4x^2)} \Gamma(\pi_L^0 \to e^+e^-)
\end{aligned} \tag{16}$$

and gives $\Gamma(\pi_L^- \to e\nu_e) \simeq 1/(3.6 \cdot 10^{-10}\ sec)$ for $m(\pi_L) = 1.062\ MeV$.

d) One must consider also the possibility that leptopion decay products are either $e_{ex}\bar{e}_{ex}$ or $e_{ex}\bar{e}$ pairs with e_{ex} having mass of near the mass of electron so that it could be misidentified as electron although the experience with the ordinary hadron physics does not give support to this possibility.

If the mass of leptonucleon e_{ex} with quantum numbers of electron is smaller than $m(\pi_L)/2$ it can be produced in leptopion annihilation. One must also assume $m(e_{ex}) > m_e$: otherwise electrons would spontaneously decay to leptonucleons via photon emission. The production rate to leptonucleon pair can be written as

$$
\begin{aligned}
\Gamma(\pi_L \to e_{ex}^+ e_{ex}^-) &= \frac{1}{\sin^4(\theta_e)} \frac{(1 - 4y^2)}{(1 - 4x^2)} \Gamma(\pi_L \to e^+ e^-) \\
y &= \frac{m(e_{ex})}{m(\pi_L)}
\end{aligned}
\tag{17}
$$

If $e - e_{ex}$ mass difference is sufficiently small the kinematic suppression does not differ significantly from that for e^+e^- pair. The limits from Babbha scattering give no bounds on the rate of $\pi_L \to e_{ex}^+ e_{ex}^-$ decay. The decay rate $\Gamma \sim 10^{26}/sec$ implied by $sin(\theta_e) \sim 10^{-4}$ implies decay width of order one $10 GeV$, which does not make sense so that the constraint $me(e_{ex}) > m(\pi_L)/2$ follows. The same argument applied to 1.8 MeV states implies the lower bound $m(e_{ex}) > .9\ MeV$.

e) The decay rate of leptosigma to $\bar{e}e_{ex}$ pair has sensible order of magnitude: for $sin(\theta_e) = 1.2 \cdot 10^{-4}$, $m_{\sigma_L} = 1.8\ MeV$ and $m_{e_{ex}} = 1.3\ MeV$ one has $\Gamma \simeq 6\ keV$ allowed by the experimental limits. This decay is kinematically possible only provided the mass of e_{ex} is in below 1.3 MeV. These decays should dominate by a factor $1/sin^2(\theta_e)$ over e^+e^- decays if kinematically allowed. A signature of these events, if identified erraneously as electron positron pairs, is the nonvanishing value of the energy difference in the cm frame of the pair: $E(e^-) - E(e^+) \simeq (m^2(e_{ex}) - m_e^2)/2E > 160\ keV$ for $E = 1.8\ MeV$. If the decay $e_{ex} \to e + \gamma$ takes place before the detection the energy asymmetry changes its sign. Energy asymmetry [Salabura et al] increasing with the rest energy of the decaying object has indeed been observed: the proposed interpretation has been that electron forms a bound state with the second nucleus so that its energy is lowered. Also a deviation from the momentum distribution implied by the decay of neutral particle to e^+e^- pair (momenta are opposite in the rest frame) results from the emission of photon. This kind of deviation has also been observed [Salabura et al]: the proposed explanation is that third object is involved in the decay. A possible alternative explanation for the asymmetries is the production mech-

anism ($\sigma_L \pi_L$ pairs instead of single particle states).

f) The decay to electron and photon would be a unique signature of e_{ex}. The general feature of of fermion family mixing is that mixing takes place in charged currents. In present case mixing is of different type so that $e_{ex} \to e + \gamma$ might be allowed. If this is not the case then the decay takes place as weak decay via the emission of virtual W boson: $e_{ex} \to e + \nu_e + \bar{\nu}_e$ and is very slow due to the presence of mixing angle and kinematical supression. The energy of the emitted photon is $E_\gamma = (m_{ex}^2 - m_e^2)/2m_e$. The decay rate $\Gamma(e_{ex} \to e + \gamma)$ is given by

$$
\begin{aligned}
\Gamma(e_{ex} \to e + \gamma) &= \alpha_{em} sin^2(\theta_e) X m_e \\
X &= \frac{(m_1 - m_e)^3 (m_1 + m_e) m_e}{(m_1^2 + m_e^2)^2 m_1}
\end{aligned}
$$

(18)

For $m(e_{ex}) = 1.3\ MeV$ the decay of order $1/(1.4 \cdot 10^{-10}\ sec)$ for $sin(\theta_e) = 1.2 \cdot 10^{-4}$ so that a considerable fraction of leptonucleons would decay to electrons in the measurement volume. In the experiments positrons are identified via pair annihilation and since pair annihilation rate for $\bar{e}_{ex}$ is by a factor $sin^2(\theta_e)$ slower than for e^+ the particles identified as positrons must indeed be positrons. For sufficiently small mass difference $m(e_{ex}) - m_e$ the particles identified as electron could actually be e_{ex}. The decay of e_{ex} to electron plus photon before its detection seems however more reasonable alternative since it could explain the observed energy asymmetry [Salabura $et\ al$].

g) The results have several implications as far as the decays of on mass shell states are considered:
i) For $m(e_{ex}) > 1.3\ MeV$ the only kinematically possible decay mode is the decay to $e^+ e^-$ pair. Production mechanism might explain the asymmetries [Salabura $et\ al$]. The decay rate of on mass shell π_L and σ_L (or $\eta_L, \rho_L, ..$) is above the lower limit allowed by the detection in the measurement volume.
ii) If the mass of e_{ex} is larger than $.9\ MeV$ but smaller than $1.3\ MeV$ $e_{ex}\bar{e}$ decays dominate over $e^+ e^-$ decays. The decay $e_{ex} \to e + \gamma$ before detection could explain the observed energy asymmetry.
iii) It will be found that the direct production of $e_{ex}\bar{e}$ pairs is also possible in the heavy ion collision but the rate is much smaller due to the smaller phase

17

space volume in two-particle case. The annihilation rate of $\bar{e}_{ex}$ in matter is by a factor $sin^2(\theta_e)$ smaller than the annihilation rate of positron. This provides a unique signature of e_{ex} if e^+ annihilation rate in matter is larger than the decay rate of $\bar{e}_{ex}$. In lead the lifetime of positron is $\tau \sim 10^{-10} \ sec$ and indeed larger that e_{ex} lifetime.

h) A brief comment on the Karmen anomaly [KARMEN] observed in the decays of π^+ is in order. The anomaly suggests the existence [Barger $et\ al$] of new weakly interacting neutral particle x, which mixes with muon neutrino. One class of solutions to laboratory constraints, which might evade also cosmological and astrophysical constraints, corresponds to object x mixing with muon type neutrino and decaying radiatively to $\gamma + \nu_\mu$ via the emission of virtual W boson. The value of the mixing parameter $U(\mu, x)$ describing $\nu_{mu} - x$ mixing satisfies $|U_{\mu,x}|^4 \simeq .8 \cdot 10^{-10}$. The following naive PCAC argument gives order of magnitude estimate for $|U(\mu, x)| \sim sin(\theta_\mu)$. The value of $g(\mu, \mu)$ is by a factor m_μ/m_e larger than $g(e, e)$. If the leptohadronic couplings $g(\mu_{ex}, \mu_{ex})$ and $g(e_{ex}, e_{ex})$ are of same order of magnitude then one has $sin(\theta_\mu) \le .02$ (3 leptopion states and Op anomaly equal to $Op = 5 \cdot 10^{-3}$): the lower bound is 6.5 times larger than the value .003 deduced in [Barger $et\ al$]. The actual value could be considerably smaller since e_{ex} mass could be larger than 1.3 MeV by a factor of order 10.

2.4   Leptopions and weak decays

The couplings of leptomeson to electroweak gauge bosons can be estimatd using PCAC and CVC hypothesis [Iztykson and Zuber]. The effective $m_{\pi_L} - W$ vertex is the matrix element of electroweak axial current between vacuum and charged leptomeson state and can be deduced using same arguments as in the case of ordinary charged pion

$$\langle 0|J_A^\alpha|\pi_L^-\rangle = Km(\pi_l)p^\alpha$$

(19)

where K is some numerical factor and p^α denotes the momentum of leptopion. For neutral leptopion the same argument gives vanishing coupling to photon by the conservation of vector current. This has the important consequence that leptopion cannot be observed as resonance in e^+e^- annihilation

in single photon channel. In two photon channel leptopion should appear as resonance. The effective interaction Lagrangian is the 'instanton' density of electromagnetic field [Pitkänen$_g$, Pitkänen and Mähonen] giving additional contribution to the divergence of the axial current and was used to derive a model for leptopion production in heavy ion collisions.

2.4.1 Leptohadrons and lepton decays

The lifetime of charged leptopion is from PCAC estimates larger than 10^{-10} seconds by the previous PCAC estimates. Therefore leptopions are practically stable particles and can appear in the final states of particle reactions. In particular, leptopion atoms are possible and by Bose statistics have the peculiar property that ground state can contain many leptopions.

Lepton decays $L \rightarrow \nu_\mu + H_L$, $L = e, \mu, \tau$ via emission of virtual W are kinematically allowed and an anomalous resonance peak in the neutrino energy spectrum at energy

$$E(\nu_L) = \frac{m(L)}{2} - \frac{m_H^2}{2m(L)} \tag{20}$$

provides a unique test for the leptohadron hypothesis. If leptopion is too light electrons would decay to charged leptopions and neutrinos unless the condition $m(\pi_L) > m_e$ holds true.

The existence of a new decay channel for muon is an obvious danger to the leptohadron scenario: large changes in muon decay rate are not allowed. a) Consider first the decay $\mu \rightarrow \nu_\mu + \pi_L$ where π_L is on mass shell leptopion. Leptopion has energy $\sim m(\mu)/2$ in muon rest system and is highly relativistic so that in the muon rest system the lifetime of leptopion is of order $\frac{m(\mu)}{2m(\pi_L)}\tau(\pi_L)$ and the average length traveled by leptopion before decay is of order 10^8 meters! This means that leptopion can be treated as stable particle. The presence of a new decay channel changes the lifetime of muon although the rate for events using $e\nu_e$ pair as signature is not changed. The effective $H_L - W$ vertex was deduced above. The rate for the decay via leptopion emission and its ratio to ordinary rate for muon decay are given by

$$\Gamma(\mu \to \nu_\mu + H_L) = \frac{G^2 K^2}{2^5 \pi} m^4(\mu) m^2(H_L)(1 - \frac{m^2(H_L)}{m^2(\mu)}) \frac{(m^2(\mu) - m^2(H_L))}{(m^2(\mu) + m^2(H_L))}$$

$$\frac{\Gamma(\mu \to \nu_\mu + H_L)}{\Gamma(\mu \to \nu_\mu + e + \bar{\nu}_e)} = 6 \cdot (2\pi^4) K^2 \frac{m^2(H_L)}{m^2(\mu)} \frac{(m^2(\mu) - m^2(H_L))}{(m^2(\mu) + m^2(H_L))}$$

$$\tag{21}$$

and is of order $.93K^2$ in case of leptopion. As far as the determination of G_F or equivalently m_W^2 from muon decay rate is considered the situation seems to be good since the change introduced to G_F is of order $\Delta G_F / G_F \simeq 0.93 K^2$ so that K must be considerably smaller than one. For the physical value of K: $K \leq 10^{-2}$ the contribution to the muon decay rate is neglibigle

Leptohadrons can appear also as virtual particles in the decay amplitude $\mu \to \nu_\mu + e\nu_e$ and this changes the value of muon decay rate. The correction is however extremely small since the decay vertex of intermediate off mass shell leptopion is proportional to its decay rate.

2.4.2 Leptopions and beta decay

If leptopions are allowed as final state particles leptopion emission provides a new channel $n \to p + \pi_L$ for beta decay of nuclei since the invariant mass of virtual W boson varies within the range ($m_e = 0.511$ $MeV, m_n - m_p = 1.293 MeV$. The resonance peak for $m(\pi_L) \simeq 1$ MeV is extremely sharp due to the long lifetime of the charged leptopion. The energy of the leptopion at resonance is

$$E(\pi_L) = (m_n - m_p)\frac{(m_n + m_p)}{2m_n} + \frac{m(\pi_L)^2}{2m_n} \simeq m_n - m_p \tag{22}$$

Together with long lifetime this leptopions escape the detector volume without decaying (the exact knowledge of the energy of charged leptopion might make possible its direct detection).

The contribution of leptopion to neutron decay rate is not negligible. Decay amplitude is proportional to superposition of divergences of axial and vector currents between proton and neutron states.

$$M = \frac{G}{\sqrt{2}} K m(\pi_L)(q^\alpha V_\alpha + q^\alpha A_\alpha) \tag{23}$$

For exactly conserved vector current the contribution of vector current vanishes identically. The matrix element of the divergence of axial vector current at small momentum transfer (approximately zero) is in good approximation given by

$$\begin{aligned} \langle p|q^\alpha A_\alpha|n\rangle &= g_A(m_p + m_n)\bar{u}_p\gamma_5 u_n \\ g_A &\simeq 1.253 \end{aligned} \tag{24}$$

Straightforward calculation shows that the ratio for the decay rate via leptopion emission and ordinary beta decay rate is in good approximation given by

$$\begin{aligned} \frac{\Gamma(n \to p + \pi_L)}{\Gamma(n \to p + e + \bar{\nu}_e)} &= \frac{30\pi^2 g_A^2 K^2}{0.47 \cdot (1 + 3g_A^2)} \frac{m_{\pi_L}^2(\Delta^2 - m_{\pi_L}^2)^2}{\Delta^6} \\ \Delta &= m(n) - m(p) \end{aligned} \tag{25}$$

Leptopion contribution is smaller than ordinary contribution if the condition

$$K < \left(\frac{.47 \cdot (1 + 3g_A^2)}{30\pi^2 g_A^2} \frac{\Delta^6}{(\Delta^2 - m_{\pi_L}^2)^2 m_{\pi_L}^2}\right)^{1/2} \simeq .28 \tag{26}$$

is satisfied. The upper bound $K \leq 10^{-2}$ coming from the leptopion decay width and Op anomaly implies that the contribution of leptopion to beta decay rate is very small.

2.5 Ortopositronium puzzle and leptopion in photon photon scattering

The decay rate of ortopositronium (Op) has been found to be slightly larger than the rate predicted by QED [Westbrook *et al*, Escribano *et al*]: the discrepancy is of order $\Delta\Gamma/\Gamma \sim 10^{-3}$. For parapositronium no anomaly has been

observed. Most of the proposed explanations [Escribano *et al*] are based on the decay mode $Op \to X + \gamma$, where X is some exotic particle. The experimental limits on the branching ratio $\Gamma(Op \to X + \gamma)$ are below the required value of order 10^{-3}. This explanation is excluded also by the standard cosmology [Escribano *et al*].

Leptopion hypothesis suggests an obvious solution of the Op-puzzle. The increase in annihilation rate is due to the additional contribution to $Op \to 3\gamma$ decay coming from the decay $Op \to \gamma_V$ (V denotes 'virtual') followed by the decay $\gamma_V \to \gamma + \pi_L^V$ followed by the decay $\pi_L^V \to \gamma + \gamma$ of the virtual leptopion to two photon state. $\gamma\gamma\pi_L$ vertices are induced by the axial current anomaly $\propto E \cdot B$. Also a modification of parapositronium decay rate is predicted. The first contribution comes from the decay $Op \to \pi_L^V \to \gamma + \gamma$ but the contribution is very small due the smallness of the coupling $g(e, e)$. The second contribution obtained from ortopositronium contribution by replacing one outgoing photon with a loop photon is also small. Since the production of real leptopion is impossible the mechanism is consistent with experimental constraints.

The modification to the Op annihilation amplitude comes in a good approximation from the interference term between the ordinary e^+e^- annihilation amplitude F_{st} and leptopion induced annihilation amplitude F_{new}:

$$\Delta\Gamma \quad \propto \quad 2Re(F_{st}\bar{F}_{new}) \tag{27}$$

and rough order of magnitude estimate suggests $\Delta\Gamma/\Gamma \sim K^2/e^2 = \alpha^2/4\pi \sim 10^{-3}$. It turns out that the sign and the order of magnitude of the new contribution are correct for $f_{\pi_L} \sim 2 \ keV$ deduced also from the anomalous e^+e^- production rate.

The new contribution to $e^+e^- \to 3\gamma$ decay amplitude is most easily derivable using for leptopion-photon interaction the effective action

$$
\begin{aligned}
L_1 &= K\pi_L F \wedge F \\
K &= \frac{\alpha_{em}}{8\pi f_{\pi_L}}
\end{aligned}
\tag{28}
$$

where F is quantized electromagnetic field. The calculation of the leptopion contribution proceeds in manner described in [Iztykson and Zuber], where

the expression for the standard contribution and an elegant method for treating the average over e^+e^- spin triplet states and sum over photon polarizations can be found. The contribution to decay rate can be written as

$$
\begin{aligned}
\frac{\Delta\Gamma}{\Gamma} &\simeq K_1 I_0 \\
K_1 &= \frac{3\alpha}{(\pi^2-9)2^9(2\pi)^3}(\frac{m_e}{f_{\pi_L}})^2 \\
I_0 &= \int_0^1 \int_{-1}^{umax} \frac{f}{v+f-1-x^2} v^2(2(f-v)u + 2 - v - f)dvdu \\
f &\equiv f(v,u) = 1 - \frac{v}{2} - \sqrt{(1-\frac{v}{2})^2 - \frac{1-v}{1-u}} \\
u &= \bar{n}_1 \cdot \bar{n}_2 \quad \bar{n}_i = \frac{\bar{k}_i}{\omega_i} \quad umax = \frac{(\frac{v}{2})^2}{(1-\frac{v}{2})^2} \\
v &= \frac{\omega_3}{m_e} \quad x = \frac{m_{\pi_L}}{2m_e}
\end{aligned}
\tag{29}
$$

ω_i and $\bar{k}_i$ denote the energies of photons, u denotes the cosine of the angle between first and second photon and v is the energy of the third photon using electron mass as unit. The condition $\Delta\Gamma/\Gamma = 10^{-3}$ gives for the parameter f_{π_L} the value $f_{\pi_L} \simeq 7.9 \ keV$. The existence of at least 3 states identifiable as σ scalars, suggests the existence of several leptopion states in one-one correspondence with sigma scalars (string model satellites). Since these states contribute to decay anomaly additively the estimate for f_{π_L} assumed to scale as m_{π_L} increases and one obtains $f_{\pi_L} \simeq 11 \ keV$ for the lightest leptopion state. From the PCAC relation one obtains for $sin(\theta_e)$ the upper bound $sin(\theta_e) \leq x \cdot 10^{-4}$ assuming $m_{ex} \geq 1.3 \ MeV$ (so that $e_{ex}\bar{e}$ decay is not possible), where $x = 1.2$ for single leptopion state and $x = 1.4$ for 3 leptopion states.

Leptopion photon interaction implies also a new contribution to photon-photon scattering. Just at the threshold $E = m_{\pi_L}/2$ the creation of leptopion in photon photon scattering is possible and the appearence of leptopion as virtual particle gives resonance type behaviour to photon photon scattering near $s = m_{\pi_L}^2$. The total photon-photon cross section in zero decay width approximation is given by

$$\sigma \quad = \quad \frac{\alpha^4}{2^{14}(2\pi)^6} \frac{E^6}{f_{\pi_L}^4 (E^2 - \frac{m_{\pi_L}^2}{4})^2} \tag{30}$$

The last column of the table 1 gives the value of the cross section at resonance.

N	$Op/10^{-3}$	f_{π_L}/keV	$sin(\theta_e)(m_{ex}/1.3\ MeV)^{1/2}$	$\Gamma(\pi_L)/keV$	$\sigma(\gamma\gamma)/\mu b$
1	1	7.9	$1.2 \cdot 10^{-4}$	.51	.03
3	1	11.0	$1.4 \cdot 10^{-4}$	.27	.007
3	5	4.9	$3.1 \cdot 10^{-4}$	1.3	.18

Table 1: The dependence of various quantities on the number of leptopion type states and Op anomaly. N refers to the number of leptopion states and Op denotes leptopion anomaly. Last column gives the value of the photon-photon scattering cross section at resonance.

2.6 Spontaneous vacuum expectation of leptopion field as source of leptopions

The basic assumption in the model of leptopion and leptohadron production is the spontaneous generation of leptopion vacuum expectation value in strong nonorthogonal electric and magnetic fields. This assumption is in fact very natural in TGD.

a) The well known relation [Iztykson and Zuber] expressing pion field as a sum of the divergence of axial vector current and anomaly term generalizes to the case of leptopion

$$\pi_L \quad = \quad \frac{1}{f_{\pi_L} m^2(\pi_L)} (\nabla \cdot j^A + \frac{\alpha_{em}}{2\pi} E \cdot B) \tag{31}$$

In the case of leptopion case the value of f_{π_L} has been already deduced from PCAC argument. Anomaly term gives rise to pion decay to two photons so that one obtains an estimate for the lifetime of the leptopion.

This relation is taken as the basis for the model describing also the production of leptopion in external electromagnetic field. The idea is that the

presence of external electromagnetic field gives rise to a vacuum expectation value of leptopion field. Vacuum expectation is obtained by assuming that the vacuum expectation value of axial vector current vanishes.

$$
\begin{aligned}
\langle vac \mid \pi \mid vac \rangle &= KE \cdot B \\
K &= \frac{\alpha_{em}}{2\pi f(\pi_L) m^2(\pi_L)}
\end{aligned}
\tag{32}
$$

Some comments concerning this hypothesis are in order here:
i) The basic hypothesis making possible to avoid large parity breaking effects in atomic and molecular physics is that p-adic condensation levels with length scale $L(n) < 10^{-6}$ m are purely electromagnetic in the sense that nuclei feed their Z^0 charges on condensate levels with $L(n) \geq 10^{-6}$ m. The absence of Z^0 charges does not however exclude the possibility of the classical Z^0 fields induced by the nonorthogonality of the ordinary electric and magnetic fields (if Z^0 fields vanish E and B are orthogonal in TGD (citeTGD).
ii) The nonvanishing vacuum expectation value of the leptopion field implies parity breaking in atomic length scales. This is understandable from basic principles of TGD since classical Z^0 field has parity breaking axial coupling to electrons and protons. The nonvanishing classical leptopion field is in fact more or less equivalent with the presence of classical Z^0 field.

b) The amplitude for the production of leptopion with four momentum $p = (p_0, \bar{p})$ in an external electromagnetic field can be deduced by writing leptopion field as sum of classical and quantum parts: $\pi_L = \pi_L(class) + \pi_L(quant)$ and by decomposing the mass term into interaction term plus c-number term and standard mass term:

$$
\begin{aligned}
\frac{m^2(\pi_L)\pi_L^2}{2} &= L_{int} + L_0 \\
L_0 &= \frac{m^2(\pi_L)}{2}(\pi_L^2(class) + \pi_L^2(quant)) \\
L_{int} &= m^2(\pi_L)\pi_L(class)\pi_L(quant)
\end{aligned}
\tag{33}
$$

Interaction Lagrangian corresponds to L_{int} linear in leptopion oscillator operators. Using standard LSZ reduction formula and normalization conventions

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of [Iztykson and Zuber] one obtains for the probability amplitude for creating leptopion of momentum p from vacuum the expression

$$
\begin{aligned}
A(p) &\equiv \langle a(p)\pi_L \rangle = (2\pi)^3 m^2(\pi_L) \int f_p(x) \langle vac \mid \pi \mid vac \rangle d^4 x \\
f_p &= e^{ip \cdot x}
\end{aligned}
\tag{34}
$$

The probability for the production of leptopion in phase space volume element $d3p$ is obtained by multiplying with the density of states factor $d^3 n = \frac{d^3 p}{(2\pi)^3 2E}$:

$$
\begin{aligned}
dP &= A \mid U \mid^2 \frac{d^3 p}{2E_p} \\
A &= (\frac{\alpha_{em}}{2\pi f(\pi_L)})^2 \\
U &= \int e^{ip \cdot x} E \cdot B d^4 x
\end{aligned}
\tag{35}
$$

The first conclusion that one can draw is that nonstatic electromagnetic fields are required for leptopion creation since in static fields energy conservation forces leptopion to have zero energy and thus prohibits real leptopion production. In particular, the spontaneous creation leptopion in static Coulombic and magnetic dipole fields of nucleus is impossible.

2.7 Sigma model and creation of leptohadrons in electromagnetic fields

2.7.1 Why sigma model approach?

For several reasons it is necessary to generalize the model for leptopion production to a model for leptohadron production.
a) Leptopions probably correspond to resonances with mass $m(\pi_L) \simeq 1.062 \, MeV$ [Chodos] and decay mostly to photon photon pairs so that $1.8 \, MeV$ resonance should correspond to some other leptomeson. Besides pseudoscalars η_L and η'_L one can consider vector bosons ρ_L and ω_L and scalar particle and its radial excitations σ_L as candidates for the observed resonances.

b) A model for the production of leptohadrons is obtained from an effective action describing the strong and electromagnetic interactions between leptohadrons. The simplest model is sigma model describing the interaction between leptonucleons, leptopion and a hypothetical scalar particle σ_L [Iztykson and Zuber]. This model realizes leptopion field as a divergence of the axial current and gives the standard relation between f_{π_L}, g and m_{ex}. All couplings of the model are related to the masses of e_{ex}, π_L and σ_L. The generation of leptopion vacuum expectation value in the proposed manner takes place via triangle anomaly diagrams in the external electromagnetic field.

c) If needed the model can be generalized to contain terms describing also other leptohadrons. The generalized model should contain also vector bosons ρ_L and ω_L as well as pseudoscalars η_L and η'_L and radial excitations of π_L and σ_L. An open question is whether also η and η' generate vacuum expectation value proportional to $E \cdot B$.

d) The following argument suggests that the most plausible identification of 1.8 MeV resonance is as σ_L so that sigma model indeed provides a satisfactory description of the situation.

i) The mass of e_{ex} must be so large that the decays to $e_{ex}e_{ex}^-$ pairs are forbidden (they would lead to nonsensically large decay width). The most plausible production mechanism for e^+e^- pairs is the decay of leptomeson to e^+e^- pair but one cannot exclude the decay to $e_{ex}\bar{e}$ and subsequent decay $e_{ex} \to e + \gamma$.

ii) Ortopositronium decay width gives $f_{\pi_L} \sim .0021\ MeV$ and from this one can deduce an upper bound for leptopion production cross section in an external electromagnetic field. The calculation of leptopion production cross section shows that leptopion production cross section is somewhat smaller than the observed e^+e^- production cross section, even when one tunes the values of the various parameters. Since pseudoscalars are expected to decay mostly to photon pairs leptopions, η_L and η'_L as main source of e^+e^- pairs are unprobable.

iii) The direct production of the pairs via the interaction term $gsin(\theta_e)\bar{e}_L\gamma_5 e_{ex}\pi_L(cl)$ from is much slower process than the production via the meson decays and does not give rise to resonant structures since mass squared spectrum for pairs forms continuum. Also the production via the $\bar{e}e_{ex}$ decay of virtual leptopion created from classical field is slow process since it involves $sin^2(\theta_e)$.

iv) e^+e^- production can proceed also via the creation of many particle states.

The simplest candidates are $V_L + \pi_L$ states created via $\partial_\alpha \pi_L V^\alpha \pi_L (class)$ term in action and $\sigma_L + \pi_L$ states created via the the $k\sigma_L \pi_L \pi_L (class)$ term in the sigma model action. The production cross section via the decays of vector mesons is certainly very small since the production vertex involves the inner product of vector boson 3 momentum with its polarization vector and the situation is nonrelativistic.

v) The pleasant surprise is that the production rate for σ_L meson is large since the coupling k turns out to be given by $k = (m_{\sigma_L}^2 - m_{\pi_L}^2)/2f_{\pi_L}$ and is anomalously large for the value of $f_{\pi_L} \geq .0079\ MeV$ derived from or-topositronium anomaly: $k \sim 336m(\pi_L)$ for $f_{\pi_L} \sim 7.9\ keV$. The resulting additional factor in the production cross section compensates the reduction factor coming from two-particle phase space volume and gives a cross section, which is rather near to the maximum value of the observed cross section.

2.7.2   Simplest sigma model

A detailed description of the sigma model can be found in [Iztykson and Zuber] and it suffices to outline only the crucial features here.

a) The action of leptohadronic sigma model reads as

$$
\begin{aligned}
L &= L_S + c\sigma_L \\
L_S &= \bar{\psi}_L(i\gamma^k\partial_k + g(\sigma_L + i\pi_L \cdot \tau\gamma_5))\psi_L + \frac{1}{2}((\partial\pi_L)^2 + (\partial\sigma_L)^2) \\
&\quad - \frac{\mu^2}{2}(\sigma_L^2 + \pi_L^2) - \frac{\lambda}{4}(\sigma_L^2 + \pi_L^2)^2
\end{aligned}
\tag{36}
$$

π_L is isospin triplet and σ_L isospin singlet. ψ_L is isospin doublet with electroweak quantum numbers of electron and neutrino (e_{ex} and ν_{ex}). The model allows $so(4)$ symmetry. Vector current is conserved but for $c \neq 0$ axial current generates divergence, which is proportional to pion field: $\partial^\alpha A_\alpha = -c\pi_L$.

b) The presence of the linear term implies that σ_L field generates vacuum expectation value $\langle 0|\sigma_L|0\rangle = v$. When the action is written in terms of new quantum field $\sigma_L' = \sigma_L - v$ one has

$$
L = \bar{\psi}_L(i\gamma^k\partial_k + m + g(\sigma_L' + i\pi_L \cdot \tau\gamma_5))\psi_L + \frac{1}{2}((\partial\pi_L)^2 + (\partial\sigma_L')^2)
$$

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$$- \frac{1}{2}m_{\sigma_L}^2 (\sigma_L')^2 - \frac{m_{\pi_L}^2}{2}\pi_L^2$$
$$- \lambda v \sigma_L'((\sigma_L')^2 + \pi_L^2) - \frac{\lambda}{4}((\sigma_L')^2 + \pi_L^2)^2$$

$$(37)$$

The masses are given by

$$
\begin{aligned}
m_{\pi_L}^2 &= \mu^2 + \lambda v^2 \\
m_{\sigma_L}^2 &= \mu^2 + 3\lambda v^2 \\
m &= -gv
\end{aligned}
$$

$$(38)$$

These formulas relate the parameters μ, v, g to leptohadrons masses.

c) The requirement that σ_L' has vanishing vacuum expectation implies in Born approximation

$$c - \mu^2 v - \lambda v^3 = 0 \qquad (39)$$

which implies

$$
\begin{aligned}
f_{\pi_L} &= -v = -\frac{c}{m^2(\pi_L)} \\
m_{ex} &= g f_{\pi_L}
\end{aligned}
$$

$$(40)$$

Note that e_{ex} and ν_{ex} are predicted to have identical masses in this approximation.

d) A new feature is the generation of the leptopion vacuum expectation value in an external electromagnetic field (of course, this is possible for the ordinary pion field, too!). The vacuum expectation is generated via the triangle anomaly diagram in a manner identical to the generation of a nonvanishing photon-photon decay amplitude and is proportional to the instanton density of the electromagnetic field. By redefining the pion field as a sum $\pi_L = \pi_L(cl) + \pi_L'$ one obtains effective action describing the creation of the leptohadrons in strong electromagnetic fields.

e) As far as the production of $\sigma_L \pi_L$ pairs is considered, the interaction term

$\lambda v \sigma'_L \pi_L^2$ is especially interesting since it leads to the creation of $\sigma_L \pi_L$ pairs via the interaction term $k \lambda v \sigma'_L \pi_L(cl)$. The coefficient of this term can be expressed in terms of the leptomeson masses and f_{π_L}:

$$k \;\equiv\; 2\lambda v = \frac{m_{\sigma_L}^2 - m_{\pi_L}^2}{2 f_{\pi_L}} \tag{41}$$

The large value of the coupling ($k \sim 336 m_{\pi_L}$ for $f_{\pi_L} = 7.9 \; keV$) compensates the reduction of the production rate coming from the smallness of two-particle phase space volume as compared with single particle-phase space volume.

2.7.3 How to generalize the sigma model approach?

The simplest sigma model containing only pion and σ particle is certainly an overidealization since three resonances at energies $1.63, 1.77$ and $1.83 \; MeV$ rather than just one have been identified (besides leptopion at $1.062 \; MeV$). This suggests a generalization of the simplest sigma model approach.

a) The production of σ particle together with some other particles is necessary in order to obtain large enough $e^+ e^-$ production cross section without ad hoc assumptions about the values of coupling constants.

b) The first, rather unprobable, possibility is that some other pseudoscalars besides π_L can be produced in association with σ_L and the decay of these states gives rise to $e_{ex}\bar{e}$ pairs since the direct decay to $e^+ e^-$ is too slow as compared with $\gamma\gamma$ decay. This requires that the mass of e_{ex} is below $1.12 \; MeV$. The pseudoscalars are probably not the leptonic counterparts of K_0, η η' meson: these pseudoscalars contain $g = 1$ color octet leptons (counterparts of strange quarks) and their masses are expected to be larger than the observed masses.

c) The second possibility is that the states with mass above $1.6 \; MeV$ correspond to radial excitations of leptosigma. In string model radial excitations correspond to satellite trajectories of the highest Regge trajectory and the states obey the mass formula

$$M^2(J,n) \;=\; M_0^2 + T(J - 2n) \tag{42}$$

Here $J = 0$ is the spin of the resonance, the integer n labels the satellite in question and T denotes leptohadronic string tension of order one MeV.

$2n$ appears instead of n in the formula to guarantee that states are scalars. The number of satellites is clearly finite. The masses of resonances above 1.6 MeV are in a satisfactory approximation evenly spaced, which suggests that the condition $T << M_0^2$ holds true. This scenario leads to the prediction of altogether 9 satellites of σ_0 with mass $\sim 1.83\ MeV$. The production rate for satellite is proportional to the factor $(m_{\sigma_L(n)}^2 - m_{\pi_L}^2)^2$ so that production probability for the lowest mass satellites is smaller and might explain why these states have not been observed.

d) Same formula predicts satellites for leptopion, too. These states have masses below $2m_e$ and can decay to two-photon states only. Stability of electron against $\pi_L + \nu$ decay implies $m_{\pi_L} \geq m_e$ and the smallness of Op anomaly as well as the experimental absence of the decay $Op \rightarrow \pi_L + \gamma$ implies $m_{\pi_l} > 2m_e$ so that 1.062 MeV state must be the lightest leptopion state. If leptopion and sigma satellites form $so(4)$ multiplets this means that π_L has at least 2 satellites. The estimate for the masses of leptopion and sigma states are given in table below for string tension $T = 0.178\ MeV$.

n	0	1	2
$m(\pi_L)/MeV$	1.36	1.22	1.062
$m(\sigma_L)/MeV$	1.83	1.73	1.62

Table 2. String model mass estimate for the satellites of π_L and σ_L.

The naivest generalization of the sigma model means the arrangement $\sigma_L - \pi_L$ pairs associated with various satellites to $so(4)$ multiplets so that each pair gives its own contribution to the sigma model action. The simplest assumption is that the sigma model couplings g and λ associated with various satellites are identical and c scales as $m_{\pi_L}^3$. It seems natural to associate to given a given meson multiplet the corresponding leptonucleon satellite: the mass of the leptonucleon would scale as m_{π_L} in the simplest scenario.

2.8 Classical model for leptopion production

The nice feature of the model (and its possible generalizations) is that the production amplitudes associated with all leptohadron production reactions in external electromagnetic field are proportional to the leptopion production amplitude and apart from phase space volume factors production cross sections are expected to be given by leptopion production cross section. There-

fore it makes sense to construct a detailed model for leptopion production despite the fact that leptopion decays probably contribute only a very small fraction to the observed e^+e^- pairs.

Angular momentum barrier makes the production of leptomesons with orbital angular momentum $L > 0$ unprobable. Therefore the observed resonances are expected to be $L = 0$ pseudoscalar states. Leptopion production has two signatures which any realistic model should reproduce.
a) Data are consistent with the assumption that states are produced at rest in cm frame.
b) The production probability has a peak in a narrow region of velocities of colling nucleus around the velocity needed to overcome Coulomb barrier in head on collision. The relative width of the velocity peak is of order $\Delta\beta/\beta \simeq \cdot 10^{-2}$ [Cowan *et al*]. In Th-Th system [Cowan *et al*] two peaks at projectile energies 5.70 MeV and 5.75 MeV per nucleon have been observed. This suggests that some kind of diffraction mechanism based on the finite size of nuclei is at work.
In this section a model treating nuclei as point like charges and nucleus-nucleus collision purely classically is developed. This model yields qualitative predictions in agreement with the signature a) but fails to reproduce the possible diffraction behaviour although one can develop argument for understanding the behaviour above Coulomb wall.

The general expression for the amplitude for creation of leptopion in external electric and magnetic fields has been already derived. Let us now specialize to the case of heavy ion collision. We consider the situation, where the scattering angle of the colliding nucleus is measured. Treating the collision completely classically we can assume that collision occurs with a well defined value of the impact parameter in a fixed scattering plane. The coordinates are chosen so that target nucleus is at rest at the origin of the coordinates and colliding nucleus moves in z-direction in y=0 plane with velocity β. The scattering angle of the scattered nucleus is denoted by α, the velocity of the lepto- pion by v and the direction angles of leptopion velocity by (θ, ϕ).

The minimum value of the impact parameter for the Coulomb collision of point like charges is given by the expression

$$
\begin{aligned}
b &= \frac{b_0 cot(\alpha/2)}{2} \\
b_0 &= \frac{2Z_1 Z_2 \alpha_{em}}{M_R \beta^2}
\end{aligned}
\tag{43}
$$

where b_0 is the expression for the distance of the closest approach in head on collision. M_R denotes the reduced mass of the nucleus-nucleus system.

To estimate the amplitude for leptopion production the following simplifying assumptions are made.

a) Nuclei can be treated as point like charges. This assumption is well motivated, when the impact parameter of the collision is larger than the critical impact parameter given by the sum of radii of the colliding nuclei:

$$
b_{cr} = R_1 + R_2
\tag{44}
$$

For scattering angles that are sufficiently large the values of the impact parameter do not satisfy the above condition in the region of the velocity peak. p-Adic considerations lead to the conclusion that nuclear condensation level corresponds to prime $p \sim 2^k$, $k = 113$ (k is prime). This suggest that nuclear radius should be replaced by the size $L(113)$ of the p-adic convergence cube associated with nucleus [Pitkänen$_b$]: $L(113) \sim 2.26 \cdot 10^{-14}$ m implies that cufott radius is $b_{cr} \sim 2L(113) \sim 5.2 \cdot 10^{-14}$ m.

b) Since the velocities are nonrelativistic (about $0.12c$) one can treat the motion of the nuclei nonrelativistically and the nonretarded electromagnetic fields associated with the exactly known classical orbits can be used. The use of classical orbit doesn't take into account recoil effect caused by leptopion production. Since the mass ratio of leptopion and the reduced mass of heavy nucleus system is of order 10^{-5} the recoil effect is however negligible.

c) The model simplifies considerably, when the orbit is idealized with a straight line with impact parameter determined from the condition expressing scattering angle in terms of the impact parameter. This approximation is certainly well founded for large values of impact parameter. For small values of impact parameter the situation is quite different and an interesting problem is whether the contributions of long range radiation fields created by

accelerating nuclei in head-on collision could give large contribution to leptopion production rate. On the line connecting the nuclei the electric part of the radiation field created by first nucleus is indeed parallel to the magnetic part of the radiation field created by second nucleus. In this approximation the instanton density in the rest frame of the target nucleus is just the scalar product of the Coulombic electric field E of the target nucleus and of the magnetic field B of the colliding nucleus obtained by boosting it from the Coulomb field of nucleus at rest.

2.8.1 Cutoff length scales in the classical model

The differential cross section in the classical model can be written as

$$
\begin{aligned}
dP &= K_0 \mid U(b) \mid^2 \frac{d^3p}{2(2\pi)^3 E_p} 2\pi b db \\
K_0 &= (\frac{\alpha_{em}}{2\pi f(\pi_L)})^2 \\
U(b,p) &= \int e^{ip\cdot x} E \cdot B d^4 x
\end{aligned}
\tag{45}
$$

where b denotes impact parameter. In the calculation of the total cross section one must introduce some cutoff radii.

Consider first the choice of the lower cutoff length scale b_{cr}.
a) Since leptopion production has maximum at energy near Coulomb wall suggest that the finite size of the colliding nuclei might play important role in the collision for the values of the scattering angle and velocity considered.
b) Lower impact parameter cutoff makes sense if the contribution of small impact parameter collisions to the production amplitude is small. This seems to be the case. For head on collision $E \cdot B$ vanishes identically and by continuity leptopion production amplitude must decrease with increasing value of scattering angle for small values of impact parameter. The value of b_{cr} should lie somehere between $2 \cdot 10^{-14}$ m and 10^{-13} m but its exact value is subject to considerable uncertainty. For impact parameters below the value of two nuclear radii point like nature of nuclei is not a good approximation and the value of $E \cdot B$ becomes more or less random in the interaction region and Fourier transform of $E \cdot B$ becomes small. For fixed scattering angle of nuclei this could explain why production rate becomes small above Coulomb

wall. What happens is that for fixed value of θ the impact parameter b becomes smaller than $b_{cr} \sim 2L(113)$, when critical value of collision velocity β is reached and $E \cdot B$ becomes random in the interaction region.

c) Alternatively, lower cufoff length scale could result from the requirement that maximum scattering angle is so small for the approximation of linear nuclear motion to make sense. Assume that the maximum scattering angle is $\theta(max) = n\theta(min)$, $\theta(min) = 2Z_1 Z_2 \alpha / M_R a \beta^2 \sim 4 \cdot 10^{-2}/A$ for $a \sim 10^{-10}$ m with $\theta(max) \sim .1$. This gives $b_{cr} \sim 10^{-13}$ m. This scale is by a factor of order two larger than that lower cutoff length scale given by the p-adic argument so that there seems to be a region of impact parameters, where the approximation of linear motion need not be good. If the contribution of the large angle collisions having $\theta \geq n\theta_1$ to the production amplitude is small then the decrease of the production probability could occur already below the Coulomb wall.

Consider next the constraints on the upper cutoff length scale.

a) The production amplitude turns out to decrease exponentially as a function of impact parameter b unless leptopion is produced in scattering plane. The contribution of leptopions produced in scattering plane however gives divergent contribution to the total cross section integrated over all impact parameter values and upper cutoff length scale a is necessary. If one considers scattering with scattering angle between specified limits this is of course not a problem of classical model.

b) Upper cutoff length scale a should be certainly smaller than the interatomic distance. A more stringent upper bound for a is the size r of atom defined as the distance above which atom looks essentially neutral: a rough extrapolation from hydrogen atom gives $r \sim a_0 / Z^{1/3} \sim 1.5 \cdot 10^{-11}$ m (a_0 is Bohr radius of hydrogen atom). Therefore cutoff scale is between Bohr radius $a_0 / Z \sim .5 \cdot 10^{-12}$ m and r.

c) One could perhaps understand the appearence of the upper cutoff length scale of order 10^{-11} m from p-adic considerations. Leptopions have primary p-adic condensation level $k = 127$ and are condensed on level $k = 131$. Leptopions are created at condensate level $k = 131$ in the classical electromagnetic fields of the colliding nuclei. $L(131)$ serves as a natural infrared cutoff for p-adic physics at leptopion condensation level $k = 131$ so that one must conclude that leptopion production rate should be calculated from p-adic physics. One can however hope that the model based on real numbers gives

satisfactory description of the situation, when the presence of the p-adic cutoff length scale is taken into account. Notice that $a = 10^{-11}$ m corresponds also to de Broglie wavelength for the leptopions of velocity $v_{cm} \sim .1$.

2.8.2 Production amplitude

The Fourier transform of $E \cdot B$ can be expressed as a convolution of Fourier transforms of E and B and the resulting expression for the amplitude reduces by residue calculus (see APPENDIX) to the following general form

$$
\begin{aligned}
U &= N(CUT_1 + CUT_2) \\
N &= \frac{i}{(2\pi)^7}
\end{aligned}
\tag{46}
$$

where nuclear charges are such that Coulomb potential is $1/r$. The contribution of the first cut for $\phi \in [0, \pi/2]$ is given by the expression

$$
\begin{aligned}
CUT_1 &= \frac{1}{2}sin(\theta)sin(\phi) \int_0^{\pi/2} exp(-\frac{cos(\psi)x}{sin(\phi_0)})A_1 d\psi \\
A_1 &= \frac{Y_1}{X_1} \\
Y_1 &= sin(\theta)cos(\phi) + iKcos(\psi) \\
X_1 &= sin^2(\theta)(sin^2(\phi) - cos^2(\psi)) + K^2 - 2iKsin(\theta)cos(\psi)cos(\phi) \\
K &= \beta\gamma(1 - \frac{v_{cm}cos(\theta)}{\beta}) \\
sin(\phi_0) &= \frac{\beta\gamma}{am(\pi_L)\gamma_1}, \quad \gamma_1 = \frac{1}{\sqrt{1-v^2}}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad v_{cm} = \frac{2v}{(1+v^2)} \\
x &= \frac{b}{a}
\end{aligned}
\tag{47}
$$

The dimensionless variable $x = b/a$ is the ratio of the impact parameter to the upper cutoff radius a.

The contribution of the second cut is given by the expression

$$
CUT_2 = \frac{1}{2}usin(\theta)sin(\phi)exp(ir_1sin(\theta)cos(\phi)x) \int_0^{\pi/2} exp(-r_2cos(\psi)x)A_2 d\psi
$$

$$
\begin{aligned}
A_2 &= \frac{Y_2}{X_2} \\
Y_2 &= sin(\theta)cos(\phi)u - 2icos(\psi)(\frac{w}{v_{cm}} + \frac{v}{\beta}sin^2(\theta)cos(2\phi)) \\
X_2 &= sin^2(\theta)(\frac{sin^2(\phi)}{\gamma^2} - u^2cos^2(\psi) + \beta^2(v^2sin^2(\theta) - \frac{2vw}{v_{cm}}cos^2(\phi))) \\
&+ \frac{w^2}{v_{cm}^2} + 2iu\beta sin(\theta)cos(\phi)(vsin^2(\theta)cos(\phi) - \frac{wcos(\psi)}{v_{cm}}) \\
u &= 1 - \beta vcos(\theta) \quad w = 1 - rcos(\theta) \\
r_1 &= \frac{\beta v\gamma}{sin(\phi_0)} \quad r_2 = \frac{\gamma}{sin(\phi_0)} \quad r = \frac{v_{cm}}{\beta} \\
x &= \frac{b}{a}
\end{aligned}
\tag{48}
$$

The denominator X_2 has no poles in the physical region and the contribution of the second cut is therefore finite. Besides this the exponential damping makes the integrand small everywhere expect in the vicinity of $cos(\Psi) = 0$ and for small values of the impact parameter.

Using the symmetries

$$
\begin{aligned}
A(p_x, -p_y) &= -A(p_x, p_y) \\
A(-p_x, -p_y) &= \bar{A}(p_x, p_y)
\end{aligned}
\tag{49}
$$

of the amplitude one can calculate the amplitude for other values of ϕ.

CUT_1 gives the singular contribution to the amplitude. The reason is that the factor X_1 appearing in denominator of cut term vanishes, when the conditions

$$
\begin{aligned}
cos(\theta) &= \frac{\beta}{v_{cm}} \\
sin(\phi) &= cos(\psi)
\end{aligned}
\tag{50}
$$

are satisfied. In forward direction this condition tells that z- component of the leptopion momentum in velocity center of mass coordinate system vanishes. In laboratory this condition means that the leptopion moves in certain cone

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defined by the value of its velocity. The condition is possible to satisfy only above the threshold $v_{cm} \geq \beta$.

For $K = 0$ the integral reduces to the form

$$CUT_1 \quad = \quad \frac{1}{2}cos(\phi)sin(\phi)\lim_{\varepsilon \to 0} \frac{\int_0^{\pi/2} exp(-\frac{cos(\psi)}{sin(\phi_0)})d\psi}{(sin^2(\phi) - cos^2\psi + i\varepsilon)}$$

(51)

One can estimate the singular part of the integral by replacing the exponent term with its value at the pole. The integral contains two parts: the first part is principal value integral and second part can be regarded as integral over a small semicircle going around the pole of integrand in upper half plane. The remaining integrations can be performed using elementary calculus and one obtains for the singular cut contribution the approximate expression

$$CUT_1 \quad \simeq \quad e^{-(b/a)(sin(\phi)/sin(\phi_0))}(\frac{ln(X)}{2} + \frac{i\pi}{2}$$

$$X \quad = \quad \frac{((1+s)^{1/2} + (1-s)^{1/2})}{((1+s)^{1/2} - (1-s)^{1/2})}$$

$$s \quad = \quad sin(\phi)$$

$$sin(\phi_0) \quad = \quad \frac{\beta\gamma}{\gamma_1 m(\pi_L)a}$$

(52)

The principal value contribution to the amplitude diverges logarithmically for $\phi = 0$ and dominates over 'pole' contribution for small values of ϕ. For finite values of impact parameter the amplitude decreases expontially as a function of ϕ.

If the singular term appearing in CUT_1 indeed gives the dominant contribution to the leptopion production one can make some conclusions concerning the properties of the production amplitude. For given leptopion cm velocity v_{cm} the production associated with the singular peak is predicted to occur mainly in the cone $cos(\theta) = \beta/v_{cm}$: in forward direction this corresponds to the vanishing of the z-component of the leptopion momentum in velocity center of mass frame. Since the values of $sin(\theta)$ are of order .1 the transversal momentum is small and production occurs almost at rest in cm frame as

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observed. In addition, the singular production cross section is concentrated in the production plane ($\phi = 0$) due to the exponential dependence of the singular production amplitude on the angle ϕ and impact parameter and the presence of the logarithmic singularity. The observed leptopion velocities are in the range $\Delta v/v \simeq 0.2$ [Cowan *et al*] and this corresponds to the angular width $\Delta\theta \simeq 34$ degrees.

These conclusions are justified by the numerical calculation of leptopion production probability $P(b)$ described in the Appendix. In the figure 2.8.2 the leptopion differential cross section $\frac{d\sigma}{d\Omega dv} = 2\pi \int P(b)bdb$ integrated over impact parameters in the range (b_{cr}, a) in U-U collision is plotted as function of scattering of direction angles (θ, ϕ) of leptopion momentum. The values of various parameters are $Z_1 = Z_2 = 92$, $a = 10^{-11}$ m, $b_{cr} = 4 \cdot 10^{-14}$ m, $(\beta, v) = (.102, .106)$ (v is cm velocity), $(\theta_0, \phi_0) = (16.0, .002)$ *degrees*. The upper cutoff has values $a = 10^{-12}$ m and $a = 10^{-11}$ m. Figures a) and c) give overall view of the differential cross section and figures b) and d) display the behaviour of the differential cross section in the singular region. For fixed value of v the cross section is peaked to momenta in scattering plane near $\theta = \theta_0$.

Figure 1: Dependence of the leptopion differential production cross section $\frac{d\sigma}{d\Omega dv}$ on angles θ and ϕ in the classical model. a) Total view for $a = 10^{-12}$ m. b) Singular region for $a = 10^{-12}$ m. c) Total view for $a = 10^{-11}$ m. d) Singular region for $a = 10^{-11}$ m. Various parameter values are given in the text.

2.8.3 Leptopion production cross section in the classical model

There are no free parameters in the model and the comparison of the predicted leptopion production cross section with the measured e^+e^- production cross section serves as a stringent test of the theory. The largest experimental value for the production cross section σ_{exp} is $\sigma_{exp}(e^+e^-) \sim 5 \cdot \mu b$ [Tsertos *et al*]. The differential production cross section is concentrated around $(\theta = \theta_0, \phi \leq n\phi_0)$, $n > 1$ (see Fig. 2.8.2).

The order of magnitude for the total classical production cross section can be estimated from

$$
\begin{aligned}
\sigma(\pi_L) &\sim 2\pi \int_{b_{cr}}^{a} P(b)bdb \\
P(b) &= KV_{ph}X(b) \\
X(b) &= \int |A|^2 d\Omega \\
K &= (Z_1 Z_2 \alpha)^2 (\frac{\alpha}{2\pi})^2 (\frac{m(\pi_L)}{f_{\pi_L}})^2 \frac{1}{2^{16}\pi^{14}} \\
f_{\pi_L} &= 7.9\ keV \\
V_{ph} &\simeq \frac{1}{6}((v_{cm} + \Delta v_{cm})^3 - v_{cm}^3) \sim 5.5 \cdot 10^{-5}
\end{aligned}
$$

$$(53)$$

$|A|^2$ is the obtained from leptopion production probablity by extracting the the coefficient K: $|A|^2$ has been estimated for single velocity v since the variation of $|A|^2$ with v is rather slow. f_{π_L} has been deduced from ortopositronium decay width. $Z_1 = Z_2 = 92$ (U-U collision) has been assumed. For phase space volume factor V_{ph} it has been assumed $v_{cm} \sim .1$ and $\Delta v_{cm} \sim 0.1 \cdot v_{cm}$. The lower impact parameter cutoff has been assumed to be $b_{cr} = 4 \cdot 10^{-14}\ m$ and upper impact parameter cutoff a is varied between $10^{-12} - 10^{-11}$ meters.

The values of classical leptopion production cross section for $a = 10^{-12}\ m$ and $a = 10^{-11}\ m$ are $.5 \cdot 10^{-5}\ \mu barn$ and $.3 \cdot 10^{-3}\ \mu barn$ respectively. Classical leptopion production cross section is by several orders of magnitude smaller than the measured e^+e^- production cross section of order 5 μb. It turns out that in quantum model constructive interference at peak for different values of impact parameter cures this disease: mechanism is analogous to quantum coherence ($|A_{coh}|^2 \propto N^2$ instead of $|A_{incoh}|^2 \propto N$).

2.9 Quantum model for leptopion production

There are good reasons for considering the quantum model. First, the leptopion production cross section is by several orders of magnitude too small in classical model. Secondly, in Th-Th collisions there are indications about the presence of two velocity peaks with separation $\delta\beta/\beta \sim 10^{-2}$ [Cowan *et al*] and this suggests that quantum mechanical diffraction effects might be in question. These effects could come from the upper and/or lower length scale cutoff and from the delocalition of the wavefunction of incoming nucleus.

2.9.1 Formulation of the quantum model

The formulation of the quantum model is based on very simple rule. In the classical model the production cross section is product of differential cross section $d\sigma = 2\pi b db$ for the incoming nucleus to scatter in a given solid angle element multiplied with the differential probability $dP(b)$ to create a leptopion. In quantum model the amplitude to create leptopion is the amplitude for incoming nucleus to scatter with given impact parameter value multiplied by the amplitude to create leptopion. The product of amplitudes is taken in x-space. For the differential production cross section and production amplitude one obtains in Born approximation the expression

$$
\begin{aligned}
d\sigma &= |f_B|^2 d\Omega \frac{d^3 p}{2E_p (2\pi)^3} \\
f_B &= -\frac{m_R}{4\pi} \int exp(i\Delta k \cdot r) V(z,b) A(b) b db dz d\phi \\
V(z,b) &= \frac{Z_1 Z_2 \alpha_{em}}{r}
\end{aligned}
\tag{54}
$$

where Δk is the momentum exchange in Coulomb scattering and a vector in the scattering plane. Effectively the Coulomb potential is replaced with the product of the Coulomb potential and leptopion production amplitude $A(b)$.

The scattering amplitude can be reduced to simpler form by using the defining integral representation of Bessel functions

$$
f_B = K_0 \int F(b) J_0(\Delta k b) A(b) b db
$$

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$$
\begin{aligned}
F(b \geq b_{cr}) &= \int dz \frac{1}{\sqrt{z^2 + b^2}} = 2ln(\frac{\sqrt{a^2 - b^2} + a}{b}) \\
K_0 &= -2\pi^2 m_R Z_1 Z_2 \alpha_{em} \\
\Delta k &= 2k sin(\frac{\alpha}{2}) \quad k = M_R \beta \\
M_R &\simeq A_R m_p \quad A_R = \frac{A_1 A_2}{A_1 + A_2}
\end{aligned}
\tag{55}
$$

where the length scale cutoffs in various integrations are not written explicitly.

The presence of the impact parameter cutoffs implies that the arguments of Bessel function is large and in a satisfactory approximation one can use in the region of physical interest the approximate trigonometric representation for Bessel functions

$$
J_0(x) \simeq \sqrt{\frac{2}{\pi x}} cos(x - \frac{\pi}{4})
\tag{56}
$$

holding true for large values of x.

2.9.2 Calculation of the leptopion production amplitude in the quantum model

The details related to the calculation of the production amplitude can be found in appendix and it suffices to describe only the general treatment here. The production amplitude of the quantum model contains integrations over the impact parameter and angle parameter ψ associated with the cut. The integrands appearing in the definition of the contributions CUT_1 and CUT_2 to the scattering amplitude have simple exponential dependence on impact parameter. The function F appearing in the definition of the scattering amplitude is a rather slow varying function as compared to the Bessel function, which allows trigonometric approximation. This motivates the division of the impact parameter range into pieces so that F can approximated with its mean value inside each piece so that integration over cutoff parameters can be performed exactly inside each piece.

CUT_1 becomes also sincular at $cos(\theta) = \beta/v_{vm}$, $cos(\psi) = sin(\phi)$. The singular contribution of the production amplitude can be extracted by putting

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$cos(\psi) = sin(\phi)$ in the arguments of the exponent functions appearing in the amplitude so that one obtains a rational function of $cos(\psi)$ and $sin(\psi)$ integrable analytically. The remaining nonsingular contribution can be integrated numerically.

2.9.3 Dominating contribution to production cross section and diffractive effects

Consider now the behaviour of the dominating singular contribution to the production amplitude depending on b via the exponent factor. This amplitude factorizes into a product

$$
\begin{aligned}
f_B(sing) &= K_0 a^2 B(\Delta k) A(sing) \\
B(\Delta k) &= \int F(ax) J_0(\Delta kax) exp(-\frac{sin(\phi)}{sin(\phi_0)}x)xdx \\
&\sim \sqrt{\frac{2}{\pi\Delta ka}} \int F(ax) cos(\Delta kax - \frac{\pi}{4}) exp(-\frac{sin(\phi)}{sin(\phi_0)}x)\sqrt{x}dx \\
x &= \frac{b}{a}
\end{aligned}
\tag{57}
$$

The factor $A(sing)$ is the analytically calculable singular and dominating part of the leptopion production amplitude (see appendix) with the exponential factor excluded. The factor B is responsible for diffractive effects. The contribution of the peak to the total production cross section is of same order of magnitude as the classical production cross section.

At the peak $\phi \sim 0$ the contribution the exponent of the production amplitude is constant at this limit one obtains product of the Fourier transform of Coulomb potential with cutoffs with the production amplitude. One can calculate the Fourier transform of the Coulomb potential analytically to obtain

$$
\begin{aligned}
f_B(sing) &\simeq 4\pi K_0 \frac{(cos(\Delta ka) - cos(\Delta kb_{cr}))}{\Delta k^2}CUT_1 \\
\Delta k &= 2M_R\beta sin(\frac{\alpha}{2})
\end{aligned}
\tag{58}
$$

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One obtains oscillatory behaviour as a function of the collision velocity in fixed angle scattering and the period of oscillation depends on scattering angle and varies in wide limits.

The relationship between scattering angle α and impact parameter in Coulomb scattering translates the impact parameter cutoffs to the scattering angle cutoffs

$$
\begin{aligned}
a &= \frac{Z_1 Z_2 \alpha_{em}}{M_R \beta^2} cot(\alpha(min)/2) \\
b_{cr} &= \frac{Z_1 Z_2 \alpha_{em}}{M_R \beta^2} cot(\alpha(max)/2)
\end{aligned}
\tag{59}
$$

This gives for the argument Δkb of the Bessel function at lower and upper cutoffs the approximate expressions

$$
\begin{aligned}
\Delta ka &\simeq \frac{2Z_1 Z_2 \alpha_{em}}{\beta} \sim \frac{124}{\beta} \\
\Delta kb_{cr} &\simeq x_0 \frac{2Z_1 Z_2 \alpha_{em}}{\beta} \sim \frac{124 x_0}{\beta}
\end{aligned}
\tag{60}
$$

The numerical values are for $Z_1 = Z_2 = 92$ (U-U collision). What is remarkable that the argument Δka at upper momentum cutoff does not depend at all on the value of the cutoff length. The resulting oscillation at minimum scattering angle is more rapid than allowed by the width of the observed peak: $\Delta\beta/\beta \sim 3 \cdot 10^{-3}$ instead of $\Delta\beta/\beta \sim 10^{-2}$: of course, the measured value need not correspond to minimum scattering angle. The oscillation associated with the lower cutoff comes from $cos(2M_R b_{cr}\beta sin(\alpha/2))$ and is slow for small scattering angles $\alpha < 1/A_R \sim 10^{-2}$. For $\alpha(max)$ the oscillation is rapid: $\delta\beta/\beta \sim 10^{-3}$.

In the total production cross section integrated over all scattering angles (or finite angular range) diffractive effects disappear. This might explain why the peak has not been observed in some experiments [Cowan *et al*].

2.9.4   Cross sections in quantum model

In figure 2.9.4 the quantity

$$Y = V_{ph}sin(\alpha)\frac{d^2\sigma}{d\Omega d\cos(\alpha)dv}$$

$$V_{ph} \simeq \frac{1}{6}((v_{cm} + \Delta v_{cm})^3 - v_{cm}^3) \sim 5.5 \cdot 10^{-5} \qquad (61)$$

having same order of magnitude as total production cross section evaluated at minimum scattering angle $\alpha(min)$ in quantum model is plotted as a function of leptopion angle variables (θ, ϕ) for U-U collision. The values of the various parameters are $Z_1 = Z_2 = 92$, $b_{cr} = 4 \cdot 10^{-14}$ m, $(\beta, v) = (.102, .106)$, $(\theta_0, \phi_0) = (16.0, .002)$ $degrees$. The upper cutoff has values $a = 10^{-12}$ m and $a = 10^{-11}$ m. Differential cross section is concentrated on small values of ϕ and has a peak at θ_0. There is however a sizable contribution from other values of θ in the cross section as the plot of differential production cross section (see Fig. 2.8.2) shows. In particular, production cross section has peak at $\theta = \pi$, whose height increases with a.

An upper bound for the total leptopion production cross section is given by $\sigma_{tot} \leq \int Y d\Omega$. Actual cross section is expected to be smaller by a numerical factor not smaller than $1/10$. The order of magnitude estimate for the leptopion production cross section in quantum model is by several orders of magnitude larger than classical cross section. The reason is the constructive interference for the contributions of various impact parameter values to the amplitude at the peak. The upper bounds are summarized in table 3 for various cases: the general order of mangitude for production cross section is one $\mu barn$.

The value of $e^+ e^-$ production cross section can be estimated as follows. $e^+ e^-$ pairs are produced from via the creation of $\sigma_L \pi_L$ pairs from vacuum and subsequent decay σ_L to $e^+ e^-$ pairs. The estimate for (or rather for the upper bound of) $\pi_L \sigma_L$ production cross section is obtained as

$$\sigma(e^+ e^-) \simeq X\sigma(\pi_L)$$

$$X = \frac{V_2}{V_1}(\frac{km_{\sigma_L}}{m_{\pi_L}^2})^2$$

$$\frac{V_2}{V_1} = V_{rel} = \frac{v_{12}^3}{3(2\pi)^2} \sim 1.1 \cdot 10^{-5}$$

45

$$\frac{k}{m_{pi_L}} = \frac{(m_\sigma^2 - m_{\pi_L}^2)}{2m_{\pi_L}f_{\pi_L}} \tag{62}$$

Here V_2/V_1 of two-particle and single particle phase space volumes. V_2 is in good approximation the product $V_1(cm)V_1(rel)$ of single particle phase space volumes associated with cm coordinate and relative coordinate and one has $V_2/V_1 \sim V_{rel} = \frac{v_{12}^3}{3(2\pi)^2)} \simeq 1.1 \cdot 10^{-5}$ if the maximum value of the relative velocity is $v_{12} \sim .1$. Situation is saved by the anomalously large value of $\sigma_L\pi_L\pi_L$ coupling constant k appearing in the production vertex $k\sigma_L\pi_L\pi_L(class)$.

The resulting upper bound for the cross section is given in table 3 in 3 cases: the actual cross section contains a numerical factor not smaller than 10. Production cross section is very sensitive to the value of f_{π_L} and Op anomaly $\Delta\Gamma/\Gamma = 5 \cdot 10^{-3}$ gives upper bound 2 μb for $a = 10^{-11}$ m, which is smaller than the experimental upper bound 5 μb. The lacking factor of order 5 could come from several sources (phase space volume, sensitive depenendence of f_{π_L} on the mass of the lightest leptopion,etc...).

It must be emphasized that the estimate is very rough (the replacement of integral over the angle α with rough upper bound, estimate for the phase space volume, the values of cutoff radii, the neglect of the velocity dependence of the production cross section, the estimate for the minimum scattering angle, ...). It seems however safe to conclude that correct value of the production cross section can be reproduced with a suitable finetuning of the cutoff length scales b_{cr} and a.

N	$Op/10^{-3}$	$\Gamma(\pi_L)/keV$	$\sigma(\pi_L)/\mu b$	$\sigma(\pi_L)/\mu b$	$\sigma(e^+e^-)/\mu b$	$\sigma(e^+e^-)/\mu b$
			$a = .01$	$a = .1$	$a = .01$	$a = .1$
1	1	.51	.13	1.4	.03	.3
3	1	.27	.07	.74	.007	.08
3	5	1.3	.34	3.7	.2	2.0

Table 3. The table summarizes leptopion lifetime and the upper bounds for leptopion and e^+e^- production cross sections for lightest leptopion. N refers to the number of leptopion states and $Op = \Delta\Gamma/\Gamma$ refers to ortopositronium decay anomaly. The values of upper cutoff length a are in units of 10^{-10} m.

Figure 2: Dependence of the quantity $P = V_{ph}sin^2(\alpha)\frac{d\sigma}{d\Omega dv dcos(\alpha)}$ on angles θ and ϕ in quantum model for cutoff cutoff angle $\alpha(min)$. a) Total view for $a = 10^{-12}$ m. b) Singular region for $a = 10^{-12}$ m. c) Total view for $a = 10^{-11}$ m. d) Singular region for $a = 10^{-11}$ m. Various parameter values are given in the text.

2.9.5 Summary

The usefulness of the modelling leptopion production is that the knowledge of leptopion production rate makes it possible to estimate also the production rates for other leptohadrons and even for many particle states consisting of leptohadrons using some effective action describing the strong interactions between leptohadrons. One can consider two basic models for leptopion production. The models contain no free parameters unless one regards cufoff length scales as such. Classical model predicts the singular production characteristics of leptopion. Quantum model predicts several velocity peaks at fixed scattering angle and the distance between the peaks of the production cross section depends sensitively on the value of the scattering angle. Production cross section depends sensitively on the value of the scattering angle for a fixed collision velocity. In both models the reduction of the leptopion production rate above Coulomb wall could be undestood as a threshold effect: for the collisions with impact parameter smaller than two times nuclear radius the production amplitude becomes very small since $E \cdot B$ is more or less random for these collisions in the interaction region. The effect is visible for fixed sufficiently large scattering angle only. e^+e^- production cross section is of the observed order of magnitude provided that e^+e^- pairs originate from the creation of $\sigma_L \pi_L$ pairs from vacuum followed by the decay $\sigma_L \to e^+e^-$: radial excitations of σ_L predicted by string model explain the appeerence of several peaks and also π_L is predicted to have lower mass states.

The proposed models are certainly overidealizations: in particular the approximation that nuclear motion is free motion fails for those values of the impact parameter, which are most important in the classical model. To improve the models one should calculate the Fourier transform of $E \cdot B$ using the fields of nuclei for classical orbits in Coulomb field rather than free motion. The second improvement is related to the more precise modelling of the situation at length scales below b_{cr}, where nuclei do not behave like point like charges. A peculiar feature of the model from the point of view of standard physics is the appeerence of the classical electromagnetic fields associated with the classical orbits of the colliding nuclei in the definition of the quantum model. This is in spirit with Quantum TGD: Quantum TGD associates a unique spacetime surface (classical history) to a given 3-surface (counterpart of quantum state).

2.10 How to observe leptonic color?

The most obvious argument against leptohadrons is that their production has not been observed in hadronic collisions. The argument is wrong. Anomalously large production of low energy e^+e^- pairs [Akesson *et al*, Barshay] in hadronic collisions has been actually observed. The most natural source for photons and e^+e^- pairs are leptohadrons. There are two possibilities for the basic production mechanism.

a) Colored leptons result directly from the decay of hadronic gluons. It might be possible to exclude this alternative by simple order of magnitude estimates.

b) Colored leptons result from the decay of virtual photons. This hypothesis is in accordance with the general idea that the QCD:s associated with different condensate levels of p-adic topological condensate do not communicate. More precisely, in TGD framework leptons and quarks correspond to different chiralities of configuration space spinors: this implies that baryon and lepton numbers are conserved exactly and therefore the stability of proton. In particular, leptons and quarks correspond to different Kac Moody representations: important difference as compared with typical unified theory, where leptons and quarks share common multiplets of the unifying group. The special feature of TGD is that there are several gluons since it is possible to associate to each Kac-Moody representation gluons, which are "irreducible" in the sense that they couple only to a single Kac Moody representation. It is clear that if the physical gluons are "irreducible" the world separates into different Kac Moody representations having their own color interactions and communicating only via electroweak and gravitational interactions. In particular, no strong interactions between leptons and hadrons occur. Since colored lepton corresponds to octet ground state of Kac-Moody representations the gluonic color coupling between ordinary lepton and colored lepton vanishes.

If this picture is correct then leptohadrons are produced only via the ordinary electroweak interactions: at higher energies via the decay of virtual photon to colored lepton pair and at low energies via the emission of leptopion by photon. Consider next various manners to observe the effects of lepton color.

a) Resonance structure in photon photon scattering and energy near leptopion mass is a unique signature of leptopion.

b) The production of leptomesons in strong classical electromagetic fields (of nuclei, for example) is one possibility. There are several important constraints for the production of leptopions in this kind of situation.

i) The scalar product $E \cdot B$ must be large. Faraway from the source region this scalar product tends to vanish: consider only Coulomb field.

ii) The region, where $E \cdot B$ has considerable size cannot be too small as compared with leptopion de Broglie wavelength (large when compared with the size of nuclei for example). If this condition doesn't hold true the plane wave appearing in Fourier amplitude is essentially constant spatially and since the fields are approximately static the Fourier component of $E \cdot B$ is expressible as a spatial divergence, which reduces to a surface integral over a surface faraway from the source region. Resulting amplitude is small since fields in faraway region have essentially vanishing $E \cdot B$.

iii) If fields are exactly static, then energy conservation prohibits leptohadron production.

c) Also the production of $e^{+}e^{-}_{ex}$ pairs in nuclear electromagnetic fields is possible although the predicted cross section is small due to the presence of two-particle phase space factor. One signature of e^{-}_{ex} is emission line accompanying the decay $e^{-}_{ex} \rightarrow e^{-} + \gamma$. The collisions of nuclei in highly ionized (perhaps astrophysical) plasmas provide a possible source of leptobaryons.

d) The interaction of quantized em field with classical electromagnetic fields is one experimental arrangement to come into mind. The simplest arrangement consisting of linearly polarized photons with energy near leptopion mass plus constant classical em field does not however work. The direct production of $\pi_L - \gamma$ pairs in rapidly varying classical electromagnetic field with frequency near leptopion mass is perhaps a more realistic possibility .

e) In the collisions of hadrons, virtual photon produced in collision can decay to two colored leptons, which in turn fragment into leptohadrons. As a result leptohadrons are produced, which in turn produce leptopions and leptosigmas decaying to photon pairs and $e^{+}e^{-}$ pairs. As already noticed, anomalous production of low energy $e^{+}e^{-}$ pairs [Akesson *et al*] in hadronic collisions has been observed.

f) $e - \nu_e$ and $e - \bar{n}u_e$ scattering at energies below one MeV provide a unique signature of leptopion. In $e - \bar{\nu}_e$ scattering π_L appears as resonance.

3 Leptohadron hypothesis and solar neutrino problem

TGD predicts two new effects, which might have some role in the understanding of the peculiarities related to solar neutrinos.

a) The existence of classical long range Z^0 fields.

b) Leptohadrons and neutrino electron interaction mediated by leptopion and leptosigma exchange (also other leptomeson exchanges are in principle possible).

These effects might provide solution to the puzzling features associated with solar neutrinos.

a) Solar neutrino deficit seems to be an established fact.

b) The values of the measured neutrino flux vary. The value measured in Homestake (neutrino energies above .8 MeV is in the range $[1/4, 1/3]$ [Davis]. The value measured in Kamiokande ($E_\nu > 7\ MeV$) is roughly $1/2$ [Hirata$_1$ *et al*] and the values measured in Gallex [Anselman *et al*] and Sage [Abazov *et al*] ($E_\nu < .42\ MeV$) are are .63 (Gallex) and .44 (Sage). Within experimental errors all measurements except Homestake are consistent with the value $J \sim 1/2$ of the solar neutrino flux.

Standard model explanations of solar neutrino deficit in terms of mixing of different neutrino families seem to be excluded since they require mass difference $|m_{\nu_\mu}^2 - m_{\nu_e}^2|$, which is much smaller than the mass difference $\sqrt{\Delta m^2} \in 0.5 - 5\ eV$ suggested by the Los Alamos experiment [Louis]. The upper bound for the value of the mixing angle is so small that mixing scenarios for standard model neutrinos are totally excluded. A potential difficulty of the models trying to solve solar neutrino problem assuming that neutrinos have a magnetic moment [Voloshin *et al* , Fugusita and Tanagita] is related to supernova physics. The magnetic moment of the neutrino implies a considerable chirality flip rate for the left handed neutrino produced in supernova. If right handed neutrinos are inert they escape the supernova immediately so that the neutrino burst from the supernova becomes shorter. The data obtained from SN1987A [Hirata$_2$ *et al*] are in accordance with the absence of right handed neutrinos. In the present case this problem is not encountered. The reason is that the temperature inside supernova is so high (hundreds of MeV:s) as compared to the mass of the leptopion that the rate

for the chirality flip by leptopion exchange is negligibly small. In TGD the anomalous magnetic moment of neutrino is expected to be of same order of magnitude as in standard model (the chirality flip mechanism proposed [Pitkänen$_g$] based on Thomas precession was based on misunderstanding).

TGD inspired solution of the solar neutrino problem relies on the classical Z^0 magnetic fields associated with the strong magnetic fields of solar convective zone (in particular magnetic fields of sunspots). What happens that the scattering of neutrinos in the classical Z^0 magnetic fields of the solar convective zone causes dispersion of the original radial neutrino flux. In classical picture Z^0 magnetic fields trap left handed component of neutrino wavepacket to circular orbit, which escapes after having transformed to right handed neutrino whereas right handed component passes through without noticing the presence of the Z^0 magnetic field. The neutrino flux received at Earth is reduced by a factor $(1 - r)$, where r tells the effective fraction of solar surface covered by magnetic structures. $r = 2/3$ gives flux $1/3$ in Homestake The model also suggests an anticorrelation of the neutrino flux with sunspots. The anticorrelation has been observed in Homestake but not in other laboratories.

In TGD context one can imagine two possible explanations for the discrepancy between different measurements:
a) Leptopion exchange implies a new contribution to neutrino electron scattering, which dominates over the standard contribution at sufficiently low energies and at sufficiently low energies implies that the effective solar neutrino flux measured using neutrino-electron scattering is larger than the actual flux. In Kamiokande the neutrino-electron scattering is used to detect neutrinos whereas all the other experimental arrangements use neutrino nucleus scattering. Therefore leptopion exchange might explain Kamiokande-Homestake discrepancy as suggested in [Pitkänen$_g$] but cannot explain the discrepancy between Homestake and other measurements based on neutrino nucleon interaction. In fact, consistency requires that leptopion contribution to the neutrino nucleus scattering should be negligible at neutrino energies $7\ MeV \leq E \leq 14\ MeV$ at which Kamiokande measurements were performed. In fact, for PCAC value of leptopion coupling this is the case but if one scales the coupling by a factor of order 3 the situation changes: this however leads to suspicuously large cross section at energies near one MeV. For neutrino laboratory energies below 1 MeV the leptopion contribution to

the scattering cross section begins to dominate (for $E_\nu = .2\ MeV$ the cross section is predicted to be 25 times larger than standard model cross section) and neutrino electron scattering via leptopion exchange provides a unique manner to test leptopion idea and possibly also to observe low energy solar neutrinos. Even a more dramatic effect is the resonance contribution to $\bar{\nu} - e$ scattering at cm energy equal to leptopion mass.

b) The classical long range Z^0 fields associated with Earth might provide an explanation for the anomalously low neutrino flux measured in Homestake. South Dakota is situated much nearer to the magnetic North Pole than the other laboratories so that magnetic and also Z^0 magnetic field is expected to be stronger there. The hypothesis explains also the anticorrelation with the solar wind noticed in the Homestake data. Solar wind pushes the magnetic field lines towards Earth and makes it stronger. The effect is largest near North pole. In fact, in van Allen belts the current created by ions trapped around field lines and rotating around the Earth, leads to a decrease of the magnetic field strength near the Equator.

To make the discussion more quantitative consider now the total cross sections for the scattering of left and right handed neutrinos on electrons at solar neutrino energies. The contribution of the standard electroweak interactions to the scattering $\nu_L^1 e_2 \to \nu_L^3 e_3$ [Okun] is given by the expression

$$
\begin{aligned}
T_{stand} &= -\sqrt{2}G\bar{e}_2\gamma_\alpha(g_L P_L + g_R P_R)e_3\bar{\nu}_1\gamma^\alpha P_L\nu_4 \\
g_L &= 1 + 2sin^2(\theta_W) \quad g_R = 2sin^2(\theta_W) \\
P_L &= \frac{1-\gamma_5}{2} \quad P_R = \frac{1+\gamma_5}{2}
\end{aligned}
$$

(63)

The contribution of the leptopion exchange (see Fig. 3) is given the expression

$$
T_{\pi_L} = -\delta^2\sqrt{2}G\bar{\nu}_1\gamma_5 e_3 \frac{m_e^2}{q^2 - m^2(\pi_L)}\bar{e}_2\gamma_5\nu_4
$$

(64)

where δ parametrizes the deviation of the leptopion coupling from PCAC value: $g = \delta g_{PCAC}$.

53

If one takes seriously the proposed model predicting two satellite trajectories for leptopion, one must over all three leptopion exchanges. In this case one has several coupling constants $g(\pi_L(i), e, \nu)$, which are assumed to be identical. One could however argue that PCAC hypothesis forces the sum of the couplings to be equal to the single leptopion PCAC value: this assumption corresponds to $\delta = 1/3$. The square of the total scattering amplitude is given by

$$
\begin{aligned}
T^2_{Kamio} &= |T_{stand}|^2 + \sum_i |T_{\pi_L(i)}|^2 + 2\sum_i Re(T_{stand}T^\dagger_{\pi_L(i)}) + \sum i \neq j T_{\pi_L(i)}\bar{T}_{\pi_L(j)} \\
|T_{stand}|^2 &= 2G^2(g_R^2(p_1 \cdot p_2)^2 + g_L^2(p_2 \cdot p_4)^2 - g_L g_R(p_1 \cdot p_4)m_e^2) \\
|T_{\pi_L}|^2 &= G^2\delta^4((p_1 \cdot p_3)^2 + (p_1 \cdot p_2)^2 - p_1 \cdot p_4(p_2 \cdot p_3))m_e^2 X(i)^2 \\
2Re(T_{stand}T^\dagger_{\pi_L}) &= 4G^2\delta^2(g_L p_1 \cdot p_4 m_e^2 - 2g_R(p_1 \cdot p_3)^2)X(i)\epsilon \\
T_{\pi_L(i)}\bar{T}_{\pi_L(j)} &= G^2\delta^4((p_1 \cdot p_3)^2 + (p_1 \cdot p_2)^2 - p_1 \cdot p_4(p_2 \cdot p_3))m_e^2 X(i)X(j) \\
X(i) &= \frac{1}{(q^2 - m^2_{\pi_L(i)})} \\
\epsilon(L) &= 1 \quad \epsilon(R) = 0
\end{aligned}
\tag{65}
$$

Interference term (parameter ϵ in the formula) is present for left handed neutrinos whereas for righthanded neutrinos scattering cross section is just the sum of leptopion and standard cross sections.

The ratio of Kamiokande and Homestake effective neutrino fluxes is simply the ratio of the corresponding cross sections given by

$$
\frac{\Phi_{eff}(Kamio)}{\Phi(Home)} = \frac{\sigma_{Kamio}}{\sigma_{stand}}
\tag{66}
$$

The ratio is plotted in Fig. 3 and 3 for $\delta = 1/3$ (ideal PCAC) , $\delta = 1$ and $\delta = 2.5$ in case of left handed neutrino. For $\delta = 2.5$ the average value of the ratio is near the value $1.5 = (1/2)/(1/3)$ deduced from experimental fluxes for $7 < E(\nu) < 14~MeV$ at which Kamiokande measurement was carried out (figure 3 c). $\delta = 2.5$ alternative is probably excluded by the rapid growth of the scattering cross section below $E_\nu \sim 1~MeV$ (figure 3 c). For $\delta = 1/3$ and $\delta = 1$ the deviation of the TGD prediction from standard model cross section is negligible (figure 3 a and b). For $\delta = 1/3$ (ideal PCAC) the value

of the effective flux is only by 2 per cent larger than than standard model prediction at $E_\nu = .2\ MeV$ (figures 3 a).

Figure 3: Detection of neutrinos by leptopion exchange. The corresponding diagram involving neutral leptopion can be neglected due to the small mass of neutrino

Figure 4: The ratio of predicted $\nu_e(L) - e$ scattering cross section and standard model cross section as a function of neutrino laboratory energy for $\delta = 1/3$, $\delta = 1$ and $\delta = 3$ (figures a), b) and c)) in energy range $.2 - 14\ MeV$.

Figure 5: The ratio of predicted $\nu_e(L) - e$ scattering cross section and standard model cross section as a function of neutrino laboratory energy for $\delta = 1/3$, $\delta = 1$ and $\delta = 3$ (figures a), b) and c)) in neutrino energy range $7 - 14\ MeV$.

4 APPENDIX

4.1 Evaluation of leptopion production amplitude

4.1.1 General form of the integral

The amplitude for leptopion production with four momentum

$$
\begin{aligned}
p &= (p_0, \bar{p}) = m\gamma_1(1, v\sin(\theta)\cos(\phi), v\sin(\theta)\sin(\phi), v\cos(\theta)) \\
\gamma_1 &= 1/(1 - v^2)^{1/2}
\end{aligned}
\tag{67}
$$

is essentially the Fourier component of the instanton density

$$
U(p) = \int e^{ip\cdot x} E \cdot B d^4 x
\tag{68}
$$

associated with the electromagnetic field of the colliding nuclei.

Coordinates are chosen so that target nucleus is at rest at the origin of coordinates and colliding nucleus moves along positive z direction in $y = 0$ plane with velocity β. The orbit is approximated with straight line with impact parameter b.

Instanton density is just the scalar product of the static electric field E of the target nucleus and magnetic field B the magnetic field associated with the colliding nucleus , which is obtained by boosting the Coulomb field of static nucleus to velocity β. The flux lines of the magnetic field rotate around the direction of the velocity of the colliding nucleus so that instanton density is indeed non vanishing.

The Fourier transforms of E and B for nuclear charge 4π giving rise to Coulomb potential $1/r$ are given by the expressions

$$
\begin{aligned}
E_i(k) &= N\delta(k_0)k_i/k^2 \\
B_i(k) &= N\delta(\gamma(k_0 - \beta k_z))k_j\varepsilon_{ijz}e^{ik_x b}/((\frac{k_z}{\gamma})^2 + k_T^2) \\
N &= \frac{1}{(2\pi)^2}
\end{aligned}
\tag{69}
$$

The normalization factor corresponds to momentum space integration measure d^4p. The Fourier transform of the instanton density can be expressed as a convolution of the Fourier transforms of E and B.

$$
\begin{aligned}
U(p) &= N_1 \int E(p-k) \cdot B(k) d^4k \\
N_1 &= \frac{1}{(2\pi)^4}
\end{aligned}
\tag{70}
$$

In the convolution the presence of two deltafunctions makes it possible to integrate over k_0 and k_z and the expression for U reduces to a two-fold integral

$$
\begin{aligned}
U(p) &= N^2 N_1 \beta\gamma \int dk_x dk_y exp(ikxb)(k_x p_y - k_y p_x)/AB \\
A &= (p_z - \frac{p_0}{\beta})^2 + p_T^2 + k_T^2 - 2k_T \cdot p_T \\
B &= k_T^2 + (\frac{p_0}{\beta\gamma})^2 \\
p_T &= (p_x, p_y)
\end{aligned}
\tag{71}
$$

To carry out the remaining integrations one can apply residy calculus.
a) k_y integral is expressed as a sum of two pole contributions
b) k_x integral is expressed as a sum of two pole contributions plus two cut contributions.

4.1.2 k_y-integration

Integration over k_y can be performed by completing the integration contour along real axis to a half circle in upper half plane (see Fig. 4.1.3).

The poles of the integrand come from the two factors A and B in denominator and are given by the expressions

$$
\begin{aligned}
k_y^1 &= i(k_x^2 + (\frac{p_0}{\beta\gamma})^2)^{1/2} \\
k_y^2 &= p_y + i((p_z - \frac{p_0}{\beta})^2 + p_x^2 + k_x^2 - 2p_x k_x)^{1/2}
\end{aligned}
\tag{72}
$$

One obtains for the amplitude an expression as a sum of two terms

$$U \; = \; 2\pi i N^2 N_1 \int e^{ik_x b}(U_1 + U_2)dk_x \tag{73}$$

corresponding to two poles in upper half plane.

The explicit expression for the first term is given by

$$
\begin{aligned}
U_1 \; &= \; RE_1 + iIM_1 \\
RE_1 \; &= \; (k_x\frac{p_0}{\beta}y - p_x re_1/2)/(re_1^2 + im_1^2) \\
IM_1 \; &= \; (-k_x p_y re_1/2K_1^{1/2} - p_x p_y K_1^{1/2})/(re_1^2 + im_1^2) \\
re_1 \; &= \; (p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2 - 2p_x k_x \\
im_1 \; &= \; -2K_1^{1/2}p_y \\
K_1 \; &= \; k_x^2 + (\frac{p_0}{\beta\gamma})^2 \tag{74}
\end{aligned}
$$

The expression for the second term is given by

$$
\begin{aligned}
U_2 \; &= \; RE_2 + iIM_2 \\
RE_2 \; &= \; -((k_x p_y - p_x p_y)p_y + p_x re_2/2)/(re_2^2 + im_2^2) \\
IM_2 \; &= \; (-(k_x p_y - p_x p_y)re_2/2K_2^{1/2} + p_x p_y K_2^{1/2})/(re_2^2 + im_2^2) \\
re_2 \; &= \; -(p_z - \frac{p_0}{\beta})^2 + (\frac{p_0}{\beta\gamma})^2 + 2p_x k_x + \frac{p_0}{\beta}y - \frac{p_0}{\beta}x \\
im_2 \; &= \; 2p_y K_2^{1/2} \\
K_2 \; &= \; (p_z - \frac{p_0}{\beta})^2 + \frac{p_0}{\beta}x + k_x^2 - 2p_x k_x \tag{75}
\end{aligned}
$$

A little inspection shows that the real parts cancel each other:$RE_1 + RE_2 = 0$. A further useful result is the identitity $im_1^2 + re_1^2 = re_2^2 + im_2^2$ and the identity $re_2 = -re_1 + 2p_y^2$.

4.1.3 k_x-integration

One cannot perform k_x-integration completely using residy calculus. The reason is that the terms IM_1 and IM_2 have cuts in complex plane. One can however reduce the integral to a sum of ple terms plus integrals over the cuts.

The poles of U_1 and U_2 come from the denominators and are in fact common for the two integrands. The explicit expressions for the pole in upper halfplane, where integrand converges exponentially are given by

$$
\begin{aligned}
re_i^2 \ + \ im_i^2 &= 0, \ i = 1, 2 \\
k_x &= (-b + i(-b^2 + 4ac)^{1/2})/2a \\
a &= 4p_T^2 \\
b &= -4((p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2)p_x \\
c &= ((p_z - \frac{p_0}{\beta})^2 + p_T^2 - (\frac{p_0}{\beta\gamma})^2)^2 + 4(\frac{p_0}{\beta\gamma})^2 p_y^2 p_y^2
\end{aligned}
\tag{76}
$$

A straightforward calculation using the previous identities shows that the contributions of IM_1 and IM_2 at pole have opposite signs and the contribution from poles vanishes identically!

The cuts associated with U_1 and U_2 come from the square root terms K_1 and K_2. The condition for the appearence of the cut is that K_1 (K_2) is real and positive. In case of K_1 this condition gives

$$
k_x \ = \ it, \ t \in (0, \frac{p_0}{\beta\gamma})
\tag{77}
$$

In case of K_2 the same condition gives

$$
k_x \ = \ p_x + it, \ t \in (0, \frac{p_0}{\beta} - p_z)
\tag{78}
$$

Both cuts are in the direction of imaginary axis.

The integral over real axis can be completed to an integral over semi-circle and this integral in turn can be expressed as a sum of two terms (see Fig. 4.1.3).

$$U = 2\pi i N^2 N_1 (CUT_1 + CUT_2) \tag{79}$$

The first term corresponds to contour, which avoids the cuts and reduces to a sum of pole contributions. Second term corresponds to the addition of the cut contributions.

In the following we shall give the exprsesions of various terms in the region $\phi \in [0, \pi/2]$. Using the symmetries

$$
\begin{aligned}
A(p_x, -p_y) &= -A(p_x, p_y) \\
A(-p_x, -p_y) &= \bar{A}(p_x, p_y)
\end{aligned}
\tag{80}
$$

of the amplitude one can calculate the amplitude for other values of ϕ.

The integration variable for cuts is the imaginary part t of complexified k_x. To get a more convenient form for cut integrals one can perform a change of the integration variable

$$
\begin{aligned}
cos(\psi) &= \frac{t}{\left(\frac{p_0}{\beta\gamma}\right)} \\
cos(\psi) &= \frac{t}{\left(\frac{p_0}{\beta} - p_z\right)} \\
\psi &\in [0, \pi/2]
\end{aligned}
\tag{81}
$$

By a painstaking calculation one verifies that the expression for the contribution of the first cut is given by

$$
\begin{aligned}
CUT_1(x) &= \frac{1}{2} sin(\theta) sin(\phi) \int_0^{\pi/2} exp\left(\frac{-cos(\psi)x}{sin(\phi_0)}\right) A_1 d\psi \tag{82} \\
A_1 &= \frac{(sin(\theta)cos(\phi) + iK cos(\psi))}{X_1} \\
X_1 &= sin^2(\theta)(sin^2(\phi) - cos^2(\psi)) + K^2 \\
&\quad - 2iK sin(\theta)cos(\psi)cos(\phi) \\
K &= \beta\gamma(1 - v_{cm}cos(\theta)/\beta)
\end{aligned}
$$

$$v_{cm} = \frac{2v}{(1+v^2)} \quad sin(\phi_0) = \frac{\beta\gamma}{bm\gamma_1}$$

$$x = \frac{b}{a} \tag{83}$$

The definitions of the various auxiliary variables are given in previous formulas.

The denominator X_1 vanishes, when the conditions

$$cos(\theta) = \frac{\beta}{v_{cm}}$$

$$sin(\phi) = cos(\psi) \tag{84}$$

hold. In forward direction the conditions express the vanishing of the z-component of the leptopion velocity in velocity cm frame as one can easily realize by noticing that condition reduces to the condition $v = \beta/2$ in nonrelativistic limit. It turns out that the contribution of first cut in fact diverges logarithmically in the limit $\phi = 0$, which corresponds to the production of leptopion with momentum in scattering plane and with direction angle $cos(\theta) = \beta/v_{cm}$.

The contribution of the second cut is given by the expression

$$CUT_2(x) = (usin(\theta)sin(\phi)/2)exp(iE_1x)) \int_0^{\pi/2} exp(-E_2x))A_2d\psi$$

$$A_2 = \frac{Y}{X_2}$$

$$Y = sin(\theta)cos(\phi)u + icos(\psi)(w/v_{cm} + (v/\beta)sin^2(\theta)(sin^2(\phi) - cos^2(\phi))$$

$$X_2 = sin^2(\theta)(\frac{sin^2(\phi)}{\gamma^2} - u^2cos^2(\psi)$$

$$+ \beta^2(v^2sin^2(\theta) - \frac{2vw}{v_{cm}})cos^2(\phi))$$

$$+ \frac{w^2}{v_{cm}^2} + 2iu\beta(vsin^2(\theta)cos(\phi) - \frac{wcos(\psi)}{v_{cm}})sin(\theta)cos(\phi)$$

$$E_1 = \frac{\gamma cos(\psi)}{sin(\phi_0)}$$

$$\begin{aligned} E_2 &= \frac{\beta\gamma v sin(\theta)cos(\phi)}{sin(\phi_0)} \\ u &= 1 - \beta v cos(\theta) \quad w = 1 - \frac{v_{cm}}{\beta}cos(\theta) \end{aligned}$$

$$(85)$$

The denominator X_2 has no poles and the contribution of the second cut is therefore finite.

Figure 6: Evaluation of k_y-integral using residy calculus.

Figure 7: Evaluation of k_x-integral using residy calculus.

4.2 Production amplitude in quantum model

The previous expressions for CUT_1 and CUT_2 as such give the production amplitude in the classical model. In quantum model the produciton amplitude can be reduced to simpler form by using the defining integral representation of Bessel functions

$$
\begin{aligned}
f_B &= K_0 \int F(b) J_0(\Delta k b) A(b) b \, db \\
F(b \geq b_{cr}) &= \int dz \frac{1}{\sqrt{z^2 + b^2}} = 2 \ln\left(\frac{\sqrt{a^2 - b^2} + a}{b}\right) \\
K_0 &= -2\pi^2 m_R Z_1 Z_2 \alpha_{em} \\
\Delta k &= 2k \sin\left(\frac{\alpha}{2}\right) \quad k = M_R \beta
\end{aligned}
\tag{86}
$$

Note that F is a rather slowly varying function of b.

The presence of the impact parameter cutoffs implies that the arguments of Bessel function is large and in a satisfactory approximation one can use in the region of physical interest the approximate trigonometric representation for Bessel functions

$$
J_0(x) \simeq \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4}\right)
\tag{87}
$$

holding true for large values of x. For the numerical treatment it is advantageous to perform the integration over impact parameters before the integration over the cut parameter ψ. One can write the following general expression for the contribution of the first cut to the production amplitude in quantum model

$$
\begin{aligned}
B_1 &= K_0 \sin(\theta) \sin(\phi) \int_0^{\pi/2} H(C_1) A_1 \, d\psi \\
H(C_1) &= \int_0^1 F(ax) J_0(\Delta k a x) \exp(-C_1 a x) x \, dx \\
&\simeq \frac{\sqrt{2}}{\sqrt{\Delta k \pi}} \int_0^1 F(ax) \exp(-C_1 a x) \cos\left(\Delta k a x - \frac{\pi}{4}\right) \sqrt{x} \, dx \\
C_1 a &= \frac{\cos(\psi)}{\sin(\phi_0)}
\end{aligned}
\tag{88}
$$

Here the definition of A_1 can be found from the defining formula for CUT_1. The corresponding expression for CUT_2 reads as

$$B_2(quant) \;=\; K_0 u sin(\theta) sin(\phi) \int_0^{\pi/2} H(C_2) A_2 d\psi$$

$$
\begin{aligned}
C_2 a &= E_1 - iE_2 \\
E_1 &= \frac{\gamma cos(\psi)}{sin(\phi_0)} \\
E_2 &= \frac{u\beta\gamma v sin(\theta) cos(\phi)}{sin(\phi_0)}
\end{aligned}
\tag{89}
$$

The definition of the function $H(C)$ is same as in previous formula. The definition of A_2 can be found from the defining formula of CUT_2.

4.3   Numerical evaluation of the production amplitudes

The numerical evaluation of the production amplitude is based on the observation that the function $G(x) = F(ax)\sqrt{x}$ appearing in the definition of $H(C)$, $C = C_1, C_2$, depends varies rather slowly in the integration range as compared to the rapidly oscillating Bessel function, which can be approximated using trigonometric functions. This motivates the division of the integration range $(x_0, 1)$ of x into pieces and the approximation of $G(x)$ with its mean value inside each piece so that the remaining rapidly varying exponent functions can be integrated exactly inside each piece. This gives the following approximate expression for the function $H(C)$

$$
\begin{aligned}
H(C) &= \frac{\sqrt{2}}{\sqrt{\Delta k \pi}} \sum_n \langle F \rangle_n (G(Ca, \Delta ka, x(n+1)) - G(Ca, \Delta ka, x(n))) \\
\langle F \rangle_n &= F((x(n) + x(n+1))/2) \\
G(C, u, x) &= \frac{1}{|-iu + C|^2}(C cos(ux) + iu sin(ux)) E(Cax) \\
E(y) &= exp(-y)
\end{aligned}
\tag{90}
$$

The precise definition of the mean value of the function F at range n is to some degree a matter of taste.

The appearence of the singularity at $(\theta = \theta_0, cos(\psi) = sin(\phi))$ in the scattering amplitude is a complication, which is avoided by calculating analytically the contribution of the singular part and numerically the remaining nonsingular part of the amplitude. The singular part of the amplitude can be defined as the amplitude obtained by putting $cos(\psi) = sin(\phi)$ (the pole of denominator X_1) in various exponential factors of the amplitude so that a rational function of $cos(\psi)$ and $sin(\psi)$ integrable analytically by elementary calculus results.

In the classical model one has the representation

$$
\begin{aligned}
CUT_1(sing, x) &= sin(\theta)sin(\phi)exp(\frac{-sin(\phi)x}{sin(\phi_0)})\int_0^{\pi/2} A_1 d\psi/2 \quad (91)\\
A_1 &= \frac{(sin(\theta)cos(\phi) + iKcos(\psi))}{X_1}\\
CUT_1(reg, x) &= CUT_1(x) - CUT_1(sing, x)\\
x &= b/a \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (92)
\end{aligned}
$$

The notations are same as in the defining formula of CUT_1.

In quantum model the corresponding replacement is $C_1(cos(\psi) \rightarrow C_1(sin(\phi) \equiv D_1$. For the exponent function E appearing in the approximate integration formula the decomposition into singular and and regular parts corresponds to the following operation

$$
\begin{aligned}
E &= exp(-C_1 ax) = E_{sing} + E_{reg}\\
E_{sing} &= exp(-D_1 ax)\\
E_{reg} &= E - E_{sing} = exp(-C_1 ax) - exp(-D_1 ax)\\
C_1 &= \frac{cos(\psi)}{sin(\phi_0)} \quad D_1 = \frac{sin(\phi)}{sin(\phi_0)}
\end{aligned}
$$

$$(93)$$

The contribution of the second cut can be estimated numerically as such in both cases.

4.4 Evaluation of the singular parts of the amplitudes

The singular parts of the amplitudes $CUT_1(sing)$ and $B_1 sing$ are rational functions of $cos(\psi)$ and the integrals over ψ can be evaluated exactly.

In the classical model the expression for $A_1(sing)$ appearing as integrand in the expression of $CUT_1(sing)$ reads as

$$
\begin{aligned}
A_1(sing) &= -\frac{1}{2\sqrt{K^2 + sin^2(\theta)}}(sin(\theta)cos(\phi)A_a + iKA_b) \\
A_a &= I_1(\beta, \pi/2) = \int_0^{\pi/2} d\psi f_1 \\
A_b &= I_2(\beta, \pi/2) = \int_0^{\pi/2} d\psi f_2 \\
f_1 &= \frac{1}{(cos(\psi) - c_1)(cos(\psi) - c_2)} \\
f_2 &= cos(\psi)f_1 \\
c_1 &= \frac{-iK cos(\phi) + sin(\phi)\sqrt{K^2 + sin^2(\theta)}}{sin(\theta)} \\
c_2 &= -\bar{c}_1
\end{aligned} \tag{94}
$$

Here c_i are the roots of the polynomial X_1 appearing in the denominator of the integrand.

In quantum model the approximate expression for the singular contribution to the production amplitude can be written as

$$
\begin{aligned}
B_1(sing) &\simeq k_1 \frac{sin(\theta)sin(\phi)}{2\sqrt{K^2 + sin^2(\theta)}} \sum_n \langle F \rangle_n (I(x(n+1)) - I(x(n))) \\
I(x) &= exp(-\frac{sin(\phi)x}{sin(\phi_0)})(sin(\theta)cos(\phi)A_a(\Delta ka, x) + iKA_b(\Delta ka, x)) \\
k_1 &= 2\pi^2 m_R Z_1 Z_2 \alpha_{em} \frac{\sqrt{2}}{\sqrt{\Delta k\pi}} sin(\phi_0)
\end{aligned} \tag{95}
$$

The expressions for the amplitudes $A_a(k, x)$ and $A_b(k, x)$ read as

$$
\begin{aligned}
A_a(k,x) &= cos(kx)I_3(k,0,\pi/2) + isin(\phi_0)ksin(kx)I_5(k,0,\pi/2) \\
A_b(k,x) &= cos(kx)I_4(k,0,\pi/2) + isin(\phi_0)ksin(kx)I_3(k,0,\pi/2) \\
I_i(k,\alpha,\beta) &= \int_\alpha^\beta f_i(k)d\psi \\
f_3(k) &= \frac{cos(\psi)}{(cos^2(\psi) + sin^2(\phi_0)k^2)}f_1(k) \\
f_4(k) &= cos(\psi)f_3(k) \\
f_5(k) &= \frac{1}{(cos^2(\psi) + sin^2(\phi_0)k^2)}f_1(k)
\end{aligned}
$$

$$(96)$$

The expressions for the integrals I_i as functions of the endpoints α and β can be written as

$$
\begin{aligned}
I_1(k,\alpha,\beta) &= I_0(c_1,\alpha,\beta) - I_0(c_2,\alpha,\beta) \\
I_2(\alpha,\beta) &= c_1 I_0(c_1,\alpha,\beta) - c_2 I_0(c_2,\alpha,\beta) \\
I_3 &= C_{34}\sum_{i=1,2,j=3,4}\frac{1}{(c_i - c_j)}(c_i I_0(c_i,\alpha,\beta) - c_j I_0(c_j,\alpha,\beta)) \\
I_4 &= C_{34}\sum_{i=1,2,j=3,4}\frac{1}{(c_i - c_j)}((c_i - c_j)(\beta - \alpha) - c_i^2 I_0(c_i,\alpha,\beta) + c_j^2 I_0(c_j,\alpha,\beta)) \\
I_5 &= C_{34}\sum_{i=1,2,j=3,4}\frac{1}{(c_i - c_j)}(I_0(c_i,\alpha,\beta) - I_0(c_j,\alpha,\beta)) \\
C_{34} &= \frac{1}{c_3 - c_4} = \frac{1}{2ikasin(\phi_0)}
\end{aligned}
$$

$$(97)$$

The parameters c_1 and c_2 are the zeros of X_1 as function of $cos(\psi)$ and c_3 and c_4 the zeros of the function $cos^2(\psi) + k^2a^2sin^2(\phi_0)$:

$$
\begin{aligned}
c_1 &= \frac{-iKcos(\phi) + sin(\phi)\sqrt{K^2 + sin^2(\theta)}}{sin(\theta)} \\
c_2 &= \frac{-iKcos(\phi) - sin(\phi)\sqrt{K^2 + sin^2(\theta)}}{sin(\theta)}
\end{aligned}
$$

$$c_3 = ikasin(\phi_0)$$
$$c_4 = -ikasin(\phi_0)$$

$$(98)$$

The basic integral $I_0(c, \alpha, \beta)$ appearing in the formulas is given by

$$
\begin{aligned}
I_0(c, \alpha, \beta) &= \int_\alpha^\beta d\psi \frac{1}{(cos(\psi) - c)} \\
&= \frac{1}{\sqrt{1 - c^2}}(f(\alpha) - f(\beta)) \\
f(x) &= ln(\frac{(1 + tan(x/2)t_0)}{(1 - tan(x/2)t_0)}) \\
t_0 &= \sqrt{\frac{1 - c}{1 + c}}
\end{aligned}
$$

$$(99)$$

From the expression of I_0 one discovers that scattering amplitude has logarithmic singularity, when the condition $tan(\alpha/2) = 1/t_0$ or $tan(\beta/2) = 1/t_0$ is satisfied and appears, when c_1 and c_2 are real. This happens at the cone $K = 0$ ($\theta = \theta_0$), when the condition

$$
\begin{aligned}
\sqrt{\frac{(1 - sin(\phi))}{(1 + sin(\phi))}} &= tan(x/2) \\
x &= \alpha \ or \ \beta
\end{aligned}
$$

$$(100)$$

holds true. The condition is satisfied for $\phi \simeq x/2$. $x = 0$ is the only interesting case and gives singularity at $\phi = 0$. In the classical case this gives logarithmic singularity in production amplitude for all scattering angles.

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